UNIVARIATE PROBABILITY DISTRIBUTIONS

Introduction:

In many situations, our interest does not lie in the outcomes of an experiment as such; we may find it more useful to describe a particular property or attribute of the outcomes of an experiment in numerical terms. For example, out of three births; our interest may be in the matter of the probabilities of the number of boys. Consider the sample space of 8 equally likely sample points

GGG	GGB	GBG	BGG
GBB	BGB	BBG	BBB

Now look at the variable **"the number of boys out of three births".** This number varies among sample points in the sample space and can take values 0,1,2,3, and it is random –given to chance.

"A random variable is an uncertain quantity whose value depends on chance". A random variable may be…

[1] Discrete if it takes only a countable number of values. For example, number of dots on two dice, number of heads in three coin tossing, number of defective items, number of boys in three births and so on.

[2] Continuous if can take on any value in an interval of numbers (i.e. its possible values are unaccountably infinite). For example, measured data on heights, weights, temperature, and time and so on.

A random variable has a probability law - a rule that assigns probabilities to different values of the random variable. This probability law - the probability assignment is called the **probability distribution** of the random variable. We usually denote the random variable by X.

Discrete Probability Distributions:

The random variable X denoting "the number of boys out of three births", we introduced in the introduction of the lesson, is a discrete random variable; so it will have a discrete probability distribution. It is easy to visualize that the random variable X is a function of sample space. We can see the correspondence of sample points with the values of the random variable as follows:

GGG (X = 0)	GGB (X = 1)	GBG (X = 1)	BGG (X = 1)
GBB (X = 2)	BGB (X = 2)	BBG (X = 2)	BBB (X = 3)

The correspondence between sample points and the value of the random variable allows us to determine the probability distribution of X as follows:

P(X=0) = 1/8	since one out of 8 equally likely points leads to X = 0
P(X=1) = 3/8	since three out of 8 equally likely points leads to X = 1
P(X=2) = 3/8	since three out of 8 equally likely points leads to X = 2
P(X=3) = 1/8	since one out of 8 equally likely points leads to X = 3

The above probability statement constitute the probability distribution of the random variable

X = number of boys in three births. We may appreciate how this probability law is obtained simply by associating values of X with sets in the sample space.

(For example, the set GGB, GBG, BGG leads to X = 1). We may write down the probability distribution of X or we may plot it graphically by means of probability Histogram or a Line chart.

Probability Distribution of the Number of Boys out of Three Births

No. of Boys X	Probability P(X)
0	1/8
1	3/8
2	3/8
3	1/8

The probability distribution of a discrete random variable X must satisfy the following two conditions:

[1] $P(X = x) \geq 0$ for all values x

[2] $\sum_{\forall x} P(X = x) = 1$

These conditions must hold because the P(X = x) values are probabilities. First condition specifies that all probabilities must be greater than or equal to zero.

For the second condition, we note that for each value x, P(x) = P(X = x) is the probability of the event that the random variable equals x. Since by definition all x means all the values the random variable X may take, and since X may take on only one value at a time, the occurrences of these values are mutually exclusive events, and one of them must take place. Therefore, the sum of all the probabilities P(X = x) must be 1.00.

Cumulative Distribution Function:

The probability distribution of a discrete random variable lists the probabilities of occurrence of different values of the random variable. We may be interested in cumulative probabilities of the random variable. That is, we may be interested in the probability that the value of the random variable is at most some value x. This is the sum of all the probabilities of the values i of X that are less than or equal to x.

The cumulative distribution function (also called cumulative probability function) F(X = x) of a discrete random variable X is

$$F(X = x) = P(X \le x) = \sum_{\text{all } i \le x} P(i)$$

For example, to find the probability of at most two boys out of three births, we have

$$F(X = 2) = P(X \le 2) = \sum_{\text{all } i \le 2} P(i)$$

$$= P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$$

Properties of Distribution Function:

Let X be a discrete random variable with distribution function F(X) and probability function P(X). The distribution function possesses the following properties:

[1] F(X) is defined for each and every value of X.

[2] $0 \le F(X) \le 1$. Since, by definition, distribution function is probability of some event and probability always lies in the range 0 and 1 both inclusive.

[3] F(X) is non-decreasing function of X

i.e. if $x_1 < x_2$ then $F(x_1) \le F(x_2)$

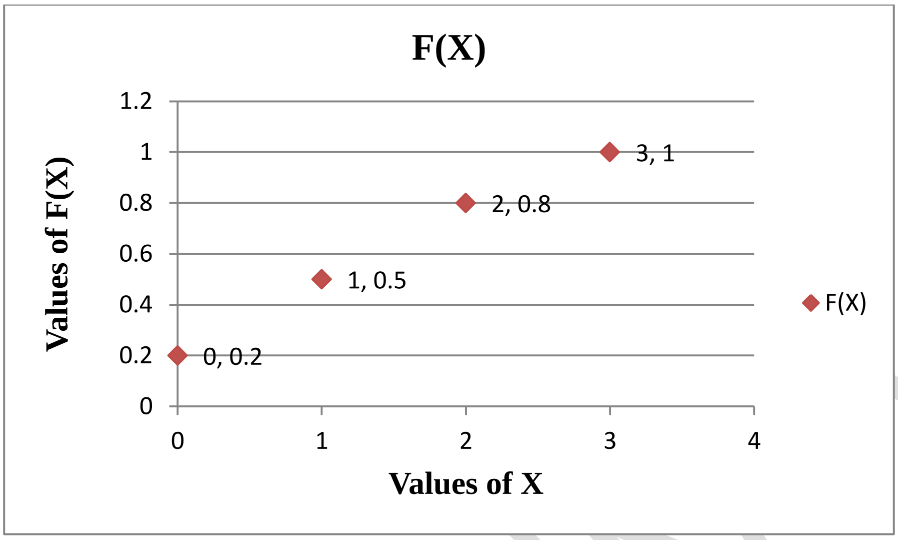
[4] $\lim\limits_{\lim \to \infty} F(x) = 1$ and $\lim\limits_{\lim \to -\infty} F(x) = 0$

[5] F(x) remains constant between two consecutive values of X and takes jumps at each value of X. The graph of distribution function looks like steps, so it is also known as a step function.

For example:

X	0	1	2	3

F(X)	0.2	0.5	0.8	1



Median of a Discrete Probability Distribution:

Let (x_i, p_i), i=1,2,...,n be probability distribution of a discrete random variable X. The median of probability distribution of X is defined as that value of X, say M such that

$$P[X \le M] \ge \frac{1}{2} \text{ and } P[X \ge M] \ge \frac{1}{2}$$

$$F(M) \ge \frac{1}{2}$$

Example:1 Determine median for the following probability distribution

X	5	10	15	20	25	30
P(X)	0.13	0.15	0.24	0.37	0.5	0.6

First find cumulative probability distribution

X	5	10	15	20	25	30
P(X)	0.13	0.15	0.24	0.37	0.5	0.6
F(X)	0.13	0.28	0.52	0.89	0.94	1

By definition of median for probability distribution

$P[X \le M] \ge \frac{1}{2}$ and $P[X \ge M] \ge \frac{1}{2}$

$F(M) \ge \frac{1}{2}$

$F[X = 15] = 0.52 > \frac{1}{2}$

Hence, Median = 15

Mode of a Discrete Probability Distribution:

Let (x_i, p_i), i=1,2,...,n be probability distribution of a discrete random variable X. The mode of probability distribution of X is defined as that value of X, say M_o for which the probability or the probability mass function (p.m.f.) is maximum.

$P[X = M_o] = \text{Maximum Probability}$

$\therefore M_o = \text{Mode}$

Example:1 Determine mode for the following probability distribution

X	5	10	15	20	25	30
P(X)	0.13	0.15	0.24	0.37	0.5	0.6

By definition of mode for probability distribution

$P[X = M_o] = \text{Maximum Probability}$

$P[X = 20] = 0.37$

Hence, Mode =20

Examples on Univariate Probability Distribution

[1] A rat is selected at random from a cage of male (M) and female rats (F). Once selected, the gender of the selected rat is noted.

The sample space is thus: S={M,F}

Define the random variable X as follows:

Let X=0 if the rat is male.

Let X=1 if the rat is female.

Note that the random variable X assigns one and only one real number (0 and 1) to each element of the sample space (M and F).

The support, or space, of X is {0,1}.

Note that we don't necessarily need to use the numbers 0 and 1 as the support.

For example, we could have alternatively (and perhaps arbitrarily?!) used the numbers 5 and 15, respectively. In that case, our random variable would be defined as X=5 of the rat is male, and X=15 if the rat is female.

[2] Let x be a random variable such that $P(x=-2) = P(x=-1)$, $P(x=2) = P(x=1), P(x > 0) = P(x < 0) = P(x = 0)$, Obtain the probability mass function of X & it's distribution function.

Answer: From the given data

X	-2	-1	0	1	2
P(X)	$\frac{x}{5}$	$\frac{x}{5}$	$\frac{2x}{5}$	$\frac{x}{5}$	$\frac{x}{5}$

As we know,

$$\sum P(x =1)=\frac{x}{5} + \frac{x}{5} + \frac{2x}{5} + \frac{x}{5}+ \frac{x}{5} = 1$$

$$\frac{6x}{5} = 1$$

$$x = \frac{5}{6}$$

X	-2	-1	0	1	2
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(ii) For probability distribution function

$$F(-2) = P(x \leq 2) = \frac{1}{6}$$

$$F(-1) = P(x \leq -1) = P(x = -2) + P(x = -1)$$

$$F(-1) = P(x \leq -1) = \frac{1}{6} + \frac{1}{6}$$

$$F(-1) = P(x \leq -1) = \frac{2}{6}$$

$$F(-1) = P(x \leq -1) = \frac{1}{3}$$

$$F(x = 0) = P(x \leq 0) = P(x = -2) + P(x = -1) + P(x = 0)$$

$$F(x = 0) = P(x \leq 0) = \frac{1}{6} + \frac{1}{6} + \frac{1}{3}$$

$$F(x = 0) = P(x \leq 0) = \frac{4}{6} = \frac{2}{3}$$

$$F(x = 1) = P(x \leq 1) = P(x = -2) + P(x = -1) + P(x = 0) + P(x = 1)$$

$$F(x = 1) = P(x \leq 1) = \frac{1}{6} + \frac{1}{6} + \frac{1}{3} + \frac{1}{6}$$

$$F(x = 1) = P(x \leq 1) = \frac{5}{6}$$

$$F(x = 2) = P(x \leq 2) =$$

$$P(x = -2) + P(x = -1) + P(x = 0) + P(x = 1) + P(x = 2)$$

$$F(x = 2) = P(x \leq 2) = \frac{1}{6} + \frac{1}{6} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6}$$

$$F(x = 2) = P(x \leq 2) = \frac{6}{6} = 1$$

Put these values in the table is

X	-2	-1	0	1	2

P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
F(X)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}=1$

[3] A random variable x assumed values -3,-2,-1, 0, 1, 2 ,3 such that $P(x=-3) = P(x=-2) = P(x=-1),\ P(x=3) = P(x=2) = P(x=1)$

$P(x = 0) = P(x > 0) = P(x < 0)$.obtain the probability mass function of x & it's distribution function.

Answer: From the given data

X	-3	-2	-1	0	1	2	3
P(X)	$\frac{x}{7}$	$\frac{x}{7}$	$\frac{x}{7}$	$\frac{2x}{7}$	$\frac{x}{7}$	$\frac{x}{7}$	$\frac{x}{7}$

As we know,

$$\sum P(x=1) = \frac{x}{7} + \frac{x}{7} + \frac{x}{7} + \frac{2x}{7} + \frac{x}{7} + \frac{x}{7} + \frac{x}{7} =$$

$$\frac{8x}{7} = 1$$

$$x = \frac{7}{8}$$

Therefore, probability distribution table is

X	-3	-2	-1	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$$F(-3) = P(x \le -3) = 1 - P(x > -3)$$

$$F(-3) = P(x \le -3) = 1 - 0 = 1$$

$$F(-2) = P(x \le -2) =$$

$$P(x \le -2) + P(x \le -1) + P(x \le 0) + P(x \le 1) + P(x \le 2) + P(x \le 3)$$

$$F(-2) = P(x \le -2) = \frac{1}{8} + \frac{1}{8} + \frac{2}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$F(-2) = P(x \le -2) = \frac{7}{8}$$

$$F(-1) = P(x \le -1) = \frac{2}{6}$$

X	-3	-2	-1	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
F(X)	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8} = 1$

[4] A random variable x has the following probability

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2k	0.3	K

(i) Find the value of k

(ii) Construct the c.d.f. f(x)

(iii) Draw it's graph

Answer: We have the probability distribution table

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2k	0.3	K

(i) We know the total probability is equal to one

$$\sum P(x = 1) = 0.1 + k + 0.2 + 2k + 0.3 + k = 1$$

0.6+ 4k = 1

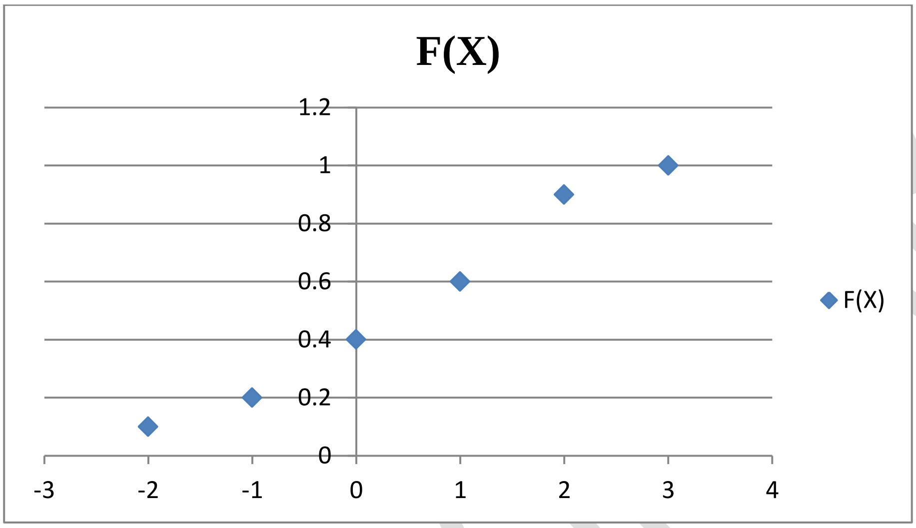
4k = 0.4

k = 0.1

(ii) Probability distribution table is

X	-2	-1	0	1	2	3
P(X)	0.1	0.1	0.2	0.2	0.3	0.1
F(X)	0.1	0.2	0.4	0.6	0.9	1.0

(iii) graph can be drown as



[5] A random variable x has the following probability distribution

X	0	1	2	3	4	5	6	7	8
P(X)	1k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value of k

(ii) Find probability of $x < 3$

(iii) Probability of $x \geq 3$

(iv) $P(0 < x < 5)$

(v) Find the distribution function of x

Answer:

(i) We know the total probability is equal to one

$\sum P(x = x) = 1$

$1k + 3k + 5k + 7k + 9k + 11k + 13k + 15k + 17k = 1$

$81k = 1$

$k = \frac{1}{81}$

Probability distribution table is

X	0	1	2	3	4	5	6	7	8
P(X)	$\frac{1}{81}$	$\frac{3}{81}$	$\frac{5}{81}$	$\frac{7}{81}$	$\frac{9}{81}$	$\frac{11}{81}$	$\frac{13}{81}$	$\frac{15}{81}$	$\frac{17}{81}$

(ii) Probability of x<3

$P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)$

$P(x < 3) = \frac{1}{81} + \frac{3}{81} + \frac{5}{81} = \frac{9}{81}$

(iii) Probability of $x \geq 3$

$P(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8)$

$P(x \geq 3) = \frac{7}{81} + \frac{9}{81} + \frac{11}{81} + \frac{13}{81} + \frac{15}{81} = \frac{72}{81}$

(iv) $P(0 < x < 5)$

$P(0 < x < 5) = P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$

$P(0 < x < 5) = \frac{3}{81} + \frac{5}{81} + \frac{7}{81} + \frac{9}{81} = \frac{24}{81}$

(v) the distribution function of x

X	0	1	2	3	4	5	6	7	8
P(X)	$\frac{1}{81}$	$\frac{3}{81}$	$\frac{5}{81}$	$\frac{7}{81}$	$\frac{9}{81}$	$\frac{11}{81}$	$\frac{13}{81}$	$\frac{15}{81}$	$\frac{17}{81}$
F(X)	$\frac{1}{81}$	$\frac{4}{81}$	$\frac{9}{81}$	$\frac{16}{81}$	$\frac{25}{81}$	$\frac{36}{81}$	$\frac{49}{81}$	$\frac{64}{81}$	$\frac{81}{81} = 1$

[6] Let P(x) is a probability function of a discrete random variable X which assumes the values x_1, x_2, x_3 and x_4 such that $2P(x_1) = 3P(x_2) = P(x_3) = 5P(x_4)$ Find probability function & cumulative distributive function of X

Answer: From the given data we can write

X	x_1	x_2	x_3	x_4
P(X)	2k	3k	K	5k

As we know that,

$\sum P(x) = 1$

$\sum P(x) = 2k + 3k + k + 5k = 1$

11k = 1

$k = \frac{1}{11}$

Put the value of k in the table to calculate probability function & probability distributive function:

X	x_1	x_2	x_3	x_4
P(X)	$\frac{2}{11}$	$\frac{3}{11}$	$\frac{1}{11}$	$\frac{5}{11}$
F(X)	$\frac{2}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{11}{11} = 1$

[7] The following is the distribution function of a discrete random variable

X	-3	-1	0	1	2	3	5	8
F(X)	0.1	0.3	0.45	0.65	0.75	0.9	0.91	1

(i) Find the probability distribution of x

(ii) Find F(X = even)=

(iii) Probability of $P(1 \le x \le 8)$

Answer:

(i) To find the probability distribution of x

$P(x = -3) = F(x = -3)$

$P(x = -1) = F(x = -3) - F(x = -1)$

$P(x = -1) = 0.3 – 0.1 = 0.2$

$P(x = 0) = F(x = 0) - F(x = -1)$

$P(x = 0) = 0.3 – 0.45 = 0.15$

$P(x = 1) = F(x = 1) - F(x = 0)$

$P(x = 1) = 0.65 – 0.15 = 0.20$

$P(x = 2) = F(x = 2) - F(x = 1)$

$P(x = 2) = 0.75 – 0.65 = 0.10$

$P(x = 3) = F(x = 3) - F(x = 2)$

$P(x = 3) = 0.9 – 0.75 = 0.15$

$P(x = 5) = F(x = 5) - F(x = 3)$

$P(x = 5) = 0.91 – 0.90 = 0.01$

$P(x = 8) = F(x = 8) - F(x = 5)$

$P(x = 8) = 1.0 – 0.91 = 0.09$

X	-3	-1	0	1	2	3	5	8
F(X)	0.1	0.3	0.45	0.65	0.75	0.9	0.91	1
P(X)	0.1	0.2	0.15	0.20	0.1	0.15	0.01	0.09

(ii) Find F(X = even) =

$F(x = \text{even}) = P(x = 0) + P(x = 2) + P(x = 8)$

$P(x = \text{even}) = 0.15 + 0.1 + 0.09 = 0.259$

(iii) Probability of $P(1 \le x \le 8)$

$P(1 \le x \le 8) = P(x = 1) + P(x = 2) + P(x = 3) + P(x = 5) + P(x = 8)$

$P(1 \le x \le 8) = 0.20 + 0.1 + 0.15 + 0.01 + 0.9 = 0.55$

[8] The probability mass function of a random variable X is 0 except at the points X = 0 , 1 , 2 at the points it has P(0) = $3c^2$, P(1) = $4c + 10c^2$, P(2) = $5c – 1$ for some $c < 0$

(i) Determine value of c

(ii) Find probability of ($x<2$)

(iii) $P(1 < x \le 2)$

(iv) Discrete the function & draw it's graph

Answer: From the above given data we can write

X	0	1	2
P(X)	$3c^2$	$4c\text{-}10c^2$	5c-1

(i) As we know,

$$\sum P(x) = 1$$

$$1 = 3c^2 + 4c - 10c^2 + 5c - 1$$

$$-7c^2 + 9c - 2 = 0$$

$$-7c^2 + 7c + 2c - 2 = 0$$

$$-7c(c-1) + 2(c-1) = 0$$

$$(c-1)(-7c+2) = 0$$

$$c - 1 = 0 \quad \text{or} \quad -7c + 2 = 0$$

$$c = 1 \quad \text{or} \quad c = \frac{2}{7}$$

Since, $c < 0$, and therefore, $c = \frac{2}{7}$

The probability distribution is

X	0	1	2
P(X)	$\frac{12}{49}$	$\frac{8}{7} - \frac{40}{49} = \frac{56-40}{49} = \frac{16}{49}$	$\frac{10}{7} - 1 = \frac{3}{7}$

(ii) To find $P(x < 2)$

$$P(x<2)=P(x=0)+P(x<1)$$

$$P(x<2)=\frac{12}{49}+\frac{16}{49}=\frac{28}{49}$$

(iii) $P(1<x\leq 2)$

$$P(1<x\ \leq 2)=P(x=2)$$

$$P(1<x\ \leq 2)=\frac{3}{7}$$

(iv) $F(x=0)=P(x=0)$

$$F(x=0)=\frac{12}{49}$$

$$F(x=1)=P(x\ \leq\ 1)$$

$$F(x=1)=P(x=0)+P(x=1)$$

$$F(x=1)=\frac{12}{49}+\frac{16}{49}=\frac{28}{49}$$

$$F(x=2)=P(x\ \leq\ 2)$$

$$F(x=2)=P(x=0)+P(x=1)+P(x=2)$$

$$F(x=2)=\frac{12}{49}+\frac{16}{49}+\frac{21}{49}=\frac{49}{49}$$

X	0	1	2
P(X)	$\frac{12}{49}$	$\frac{16}{49}$	$\frac{3}{7}$
F(X)	$\frac{12}{49}$	$\frac{28}{49}$	1

[9] Suppose that the random variable x assumes 3 values 0, 1, 2 with probability 1/3, 1/6, 1/2 respectively. Obtain the distribution function of x.

Answer: From the above given data we can write in tabular form

X	0	1	2
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$

Distribution function values are

$F(x=0) = P(x=0)$

$F(x=0) = \frac{1}{3}$

$F(x=1) = P(x \le 1)$

$F(x=1) = P(x=0) + P(x=1)$

$F(x=1) = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$

$F(x=2) = P(x \le 2)$

$F(x=2) = P(x=0) + P(x=1) + P(x=2)$

$F(x=2) = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = \frac{6}{6} = 1$

X	0	1	2
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$
F(X)	$\frac{1}{3}$	$\frac{3}{6}$	$\frac{6}{6} = 1$

[10] Given that, $f(x) = k\left(\frac{1}{2}\right)^x$ is a probability distribution for a random variable which can take the values x= 0, 1, 2, 3, 4, 5, 6 find k and c. d. f.

Answer:

$P(x) = k\left(\frac{1}{2}\right)^x \qquad x = 0, 1, 2, 3, 4, 5, 6$

$P(0) = k\left(\frac{1}{2}\right)^0 = k$

$P(1) = k\left(\frac{1}{2}\right)^1 = \frac{k}{2}$

$P(2) = k\left(\frac{1}{2}\right)^2 = \frac{k}{4}$

$P(3) = k\left(\frac{1}{2}\right)^3 = \frac{k}{8}$

$$P(4) = k\left(\frac{1}{2}\right)^4 = \frac{k}{16}$$

$$P(5) = k\left(\frac{1}{2}\right)^5 = \frac{k}{32}$$

$$P(6) = k\left(\frac{1}{2}\right)^6 = \frac{k}{64}$$

X	0	1	2	3	4	5	6
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$	$\frac{k}{16}$	$\frac{k}{32}$	$\frac{k}{64}$

We know the total probability is equal to one

$$\sum P(X) = 1$$

$$k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} + \frac{k}{16} + \frac{k}{32} + \frac{k}{64} = 1$$

$$\frac{127k}{64} = 1$$

$$k = \frac{64}{127}$$

X	0	1	2	3	4	5	6
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$	$\frac{k}{16}$	$\frac{k}{32}$	$\frac{k}{64}$
P(X)	$\frac{64}{127}$	$\frac{64}{254}$	$\frac{64}{508}$	$\frac{64}{1016}$	$\frac{64}{2032}$	$\frac{64}{4064}$	$\frac{64}{8128}$

[11] Suppose that the random variable x has possible values 1 , 2 , 3, ... and probability of $P(x) = \frac{1}{2^j}$, j = 1 ,2, 3,...,i…,n

(i) Compute probability of x- even

(ii) $P(x \geq 5)$

(iii) P(x/3)

Answer:- we know,

X has possible values 1 , 2 , 3 , ...,i ...n

$$P(x)= \frac{1}{2^j}, \quad j = 1 ,2, 3,...,i\dots,n$$

$$P(x=1) = \frac{1}{2^1}= \frac{1}{2} = 0.5$$ `

$$P(x=2) = \frac{1}{2^2}= \frac{1}{4} = 0.25$$

$$P(x=3) = \frac{1}{2^3}= \frac{1}{8}= 0.125$$

$$P(x=4) = \frac{1}{2^4}= \frac{1}{16}= 0.0625$$

$$P(x=5) = \frac{1}{2^5}= \frac{1}{32}= 0.03125$$ `

$$P(x=6) = \frac{1}{2^6}= \frac{1}{64}= 0.015625$$

$$P(x=7) = \frac{1}{2^7}= \frac{1}{128}= 0.0078125$$

$$P(x=8) = \frac{1}{2^8}= \frac{1}{256}= 0.00390625$$

$$P(x=9) = \frac{1}{2^9}= \frac{1}{512}= 0.001953125$$

$$P(x=10) = \frac{1}{2^{10}}= \frac{1}{1024}= 0.000976562$$

$$P(x=11) = \frac{1}{2^{11}}= \frac{1}{2048}= 0.000488281$$

(i) $P(x = \text{even}) = P(x = 2) + P(x = 4) + P(x = 8) + P(x = 8)$

$P(x = \text{even}) = 0.25+ 0.0625 + 0.015625 +0.0039$

$P(x = \text{even}) = 0.3320$

(ii) $P(x \geq 5) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$P(x \geq 5) = 0.5 + 0.25 + 0.125 + 0.0625 + 0.03125$

$P(x \geq 5) = 0.96$

(iii) $P(X=3x) = P(x=3) + P(x=6) + P(x=9)$

$P(X=3x) = 0.125 + 0.015625 + 0.00097$

$P(X=3x) = 0.141$

Previous Year Questions in University Exam:

[1] A random variable X has the following probability distribution: (**April-2015**)

X	0	1	2	3	4
P(X)	0.15	0.15	0.30	0.25	0.15

[2] Determine k such that the following function is a p.m.f.:

$$P(X=x) = k(x^2+2x+1) \quad ; x: 0,1,2,3$$
$$= 0 \quad ; \text{otherwise}$$

Also find $P(X=1/X \leq 2)$

[3] The cumulative distribution function c. d. f. of a discrete r. v. X is given below:

X	1	2	3	4	5	6
P(X)	0.15	0.35	0.45	0.68	0.86	1

Find:

(A) The probability distribution of r. v. X.

(B) $P[X=5/X \geq 3]$

(C) The values of median and mode of the distribution

[4] For a probability distribution of X, obtain cumulative distribution function c. d. f. of X.

X	-1	0	1
P(X)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

[5] Define moment generating function of a r. v. X. State the uniqueness property of m.g.f. (**April-2017**)