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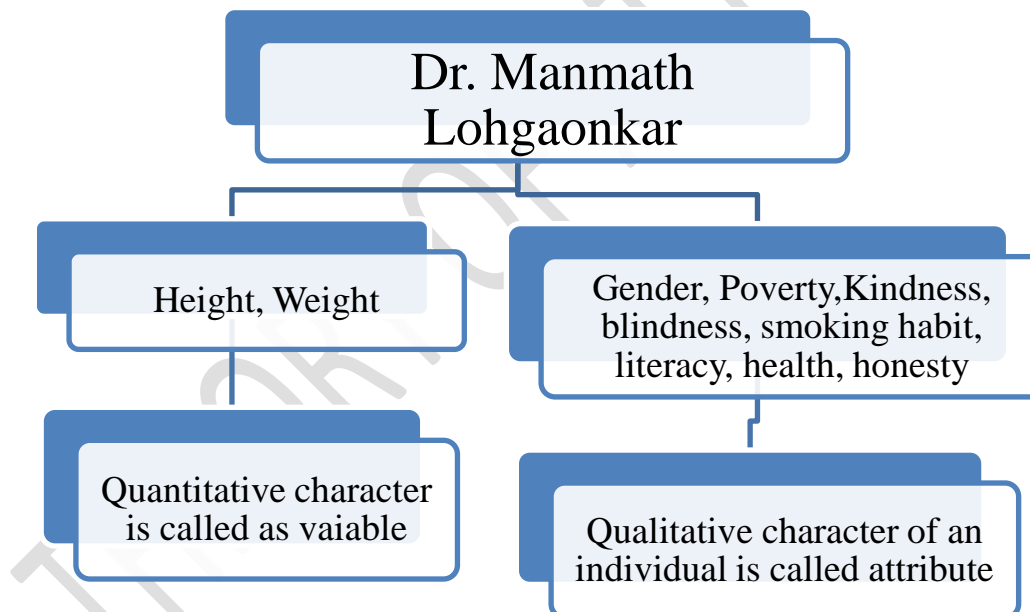
# THEORY OF ATTRIBUTES

## 5.1.1 Introduction:

Generally Statistics deals with quantitative data but in behavioural sciences data is other than quantitative .

An attribute is a characteristic of an individual which cannot be measured numerically. **For example:** Marital status, mother tongue and defectiveness of a unit of product these are attributes. Attribute may belong to one or more than two classes. whether an individual is a non-smoker, a light smoker, a heavy smoker or a chain smoker.

Dr. Manmath Lohgaonkar is an individual. Every individual has some character.



### Definition of Attribute:

Qualitative characteristic of an individual is called attribute. It is denoted by capital letters A, B, C and their absence by Greek letters  $\alpha, \beta, \gamma$ .

**For example:** Gender, Poverty, Nationality, Mother tongue, Kindness, blindness, smoking habit, literacy.

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### 5.1.2 Concept of Likert's Scale:

A Likert Scale is a type of rating scale used to measure attitude or opinions. With this scale, respondents are asked to rate items on a level of agreement.

**For example:** Strongly agree, Agree, Neutral, Disagree, Strongly disagree  
Five to seven items are usually used in the scale.

### 5.1.3 Classification

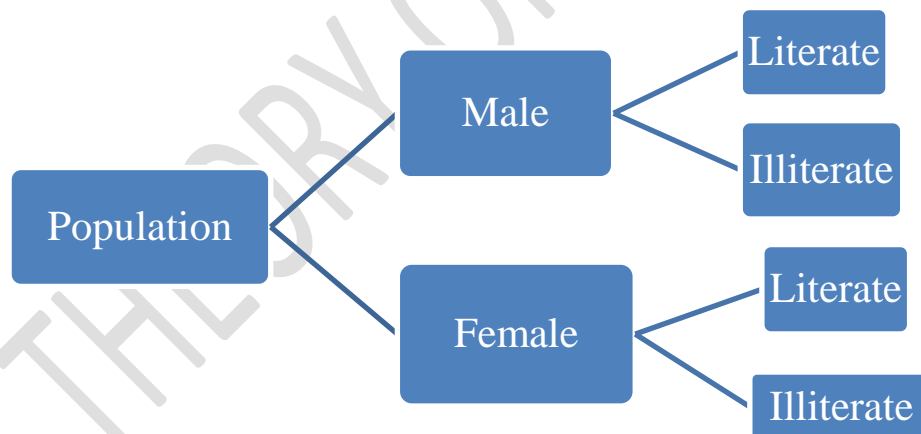
Attribute is classified into two categories. The presence of an attribute and the absence of an attribute. The total population is classified into either blind or sighted, either literate or illiterate.

#### Dichotomous classification:-

If only one attribute is studied, the population is divided into two classes according to its presence or absence and such classification is termed as dichotomous classification.

**OR**

Grouping into two at each level is called dichotomous classification.



#### 5.1.4 Manifold classification:-

If a class is divided into more than two classes such classification is termed as manifold classification.

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## Notation

Two categories should be represented by symbols. The Roman capitals can be used to represent the presence of any category, and Greek smalls used in favours of absence of that category.

Suppose if 'A' represents the 'literacy' then ' $\alpha$ ' represents the absence of 'literacy'. In a similar way if another character 'smoking habit' be represented by 'B' and ' $\beta$ ' represents the character 'non-smoking', so the combination of two characters can easily be classified in the following way:

AB - literate smokers

A $\beta$  - literate non-smokers

$\alpha$ B - illiterate smokers

$\alpha\beta$  - illiterate non-smokers

From the above notations it is clear that the characters A and  $\alpha$  are complementary to each other. Similarly, B and  $\beta$  are complementary to each other. In further the third character possessing SEBC/OBC/NT/SC/ST certificate be represented by C and not holding that be represented by  $\gamma$ , then we in similar argument can interpret as follow the combination of ABC or  $\alpha\beta\gamma$  as following

ABC - Literate smoker within SEBC/OBC/NT/SC/ST group

AB $\gamma$  - Literate smoker from other than SEBC/OBC/NT/SC/ST

A $\beta$ C - Literate non-smoker within SEBC/OBC/NT/SC/ST group

$\alpha\beta\gamma$  - Illiterate non-smokers from other than SEBC/OBC/NT/SC/ST

### 5.1.5 Class frequency:-

**Class:** Each attribute forms a group or class.

Male is an attribute and therefore it is a class. It is denoted by A, AB, A $\beta$ C.

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**Class frequency:** The number of observation within a particular class is known as the frequency of that class. It is denoted through bracket. As the frequency of class A is denoted by (A), similarly the frequencies of AB or  $\alpha$  by (AB) or ( $\alpha$ ) respectively.

**5.1.6 Order of Class:** Combination of a class of n attributes is called as n<sup>th</sup> order class **or order will depend on the number of classes involved in the grouping.**

Therefore, A, B,  $\alpha$ ,  $\beta$  etc. are first order classes. AB,  $\alpha\beta$  are second order classes. ABC,  $\alpha\beta\gamma$  are third order classes

**5.1.7 Positive class frequency:-** Class frequencies of type (A), (B), (AB) are known as positive frequencies.

**5.1.8 Negative class frequency:-** Class frequencies of type ( $\alpha$ ) ( $\alpha\beta$ ) ( $\alpha\beta\gamma$ ) are called negative frequencies.

**Contrary class frequencies:-**Class frequencies of the type ( $\alpha$ B) ( $A\beta$ ) ( $A\beta C$ ), etc. are called contrary frequencies.

**5.1.9 Ultimate class frequencies:-**

The frequencies of the classes of **highest order** are called as ultimate class frequencies.

For one attribute there will be  $2^1 = 2$  ultimate class frequencies

For two attribute there will be  $2^2 = 4$  ultimate class frequencies

For three attribute there will be  $2^3 = 8$  ultimate class frequencies

For n attribute there will be  $2^n$  ultimate class frequencies

**Number of Class frequencies:-**

For a single attribute A, there are three classes such as A,  $\alpha$  and N. N, being the total frequencies which is taken as a class.

For double attributes A and B, the number of class will be nine such as N, A, B,  $\alpha$ ,  $\beta$ , AB,  $A\beta$ ,  $\alpha B$ ,  $\alpha\beta$ . **In this way if the number of attributes be n, the total number of class frequencies will be  $3^n$ .**

For more clarity the following tables are given. Three classes A, B, C and their complementary  $\alpha$ ,  $\beta$ ,  $\gamma$  are used.

Order	Class frequencies	No. of classes	Total No. of class frequencies	Ultimate class frequencies
0	N	1		
1	(A),(B),(C),( $\alpha$ ),( $\beta$ ),( $\gamma$ )	6		
2	(AB),(A $\beta$ ),(AC),(A $\gamma$ ), (BC),(B $\gamma$ ), ( $\alpha$ B),( $\alpha$ C), ( $\beta$ C),( $\alpha\beta$ ),( $\alpha\gamma$ ),( $\beta\gamma$ )	12		
3	(ABC),(AB $\gamma$ ),( $\alpha$ BC) ( $\alpha$ B $\gamma$ ), (A $\beta$ C),( $\alpha\beta$ C), (A $\beta\gamma$ ),( $\alpha\beta\gamma$ )	8		8
	Total	27	27	

### 5.1.10 Relationship among different class frequencies:

#### Class frequency of two attributes:

	A	$\alpha$	Total
B	(AB )	( $\alpha$ B)	(B)
$\beta$	(A $\beta$ )	( $\alpha\beta$ )	( $\beta$ )
Total	(A)	( $\alpha$ )	N

The frequency of a lower order class can always be expressed in terms of the higher order class frequencies.

$$N = (A) + (\alpha) = (B) + (\beta)$$

$$(A) = (AB) + (A\beta)$$

$$(\alpha) = (\alpha B) + (\alpha\beta)$$

$$(B) = (AB) + (\alpha B)$$

$$(\beta) = (A\beta) + (\alpha\beta)$$

In the above 2 x 2 contingency table. The number of attributes are 2, then number of classes are  $3^2 = 9$  and we have  $2^2 = 4$  cell frequencies. Therefore, **if the number of attributes are n, then there will be  $3^n$  classes and we have  $2^n$  cell frequencies.**

**Class frequency of three attributes:**

	A			$\alpha$			Total		
	B	$\beta$	Total	B	$\beta$	Total	B	$\beta$	Total
C	(ABC)	(A $\beta$ C)	(AC)	( $\alpha$ BC)	( $\alpha\beta$ C)	( $\alpha$ C)	(BC)	( $\beta$ C)	(C)
$\gamma$	(AB $\gamma$ )	(A $\beta\gamma$ )	(A $\gamma$ )	( $\alpha$ B $\gamma$ )	( $\alpha\beta\gamma$ )	( $\alpha\gamma$ )	(B $\gamma$ )	( $\beta\gamma$ )	( $\gamma$ )
Total	(AB)	(A $\beta$ )	(A)	( $\alpha$ B)	( $\alpha\beta$ )	( $\alpha$ )	(B)	( $\beta$ )	N

$$(AB) = (ABC) + (AB\gamma)$$

$$(A\beta) = (A\beta C) + (A\beta\gamma)$$

$$(AC) = (ABC) + (A\beta C)$$

$$(BC) = (ABC) + (\alpha BC)$$

$$(A) = (AB) + (A\beta)$$

### 5.1.11 Fundamental set of class frequencies:

An important property of the ultimate frequencies is that the frequency of any class can be expressed in terms of these. Thus we have

$$(A) = (ABC) + (AB\gamma) + (A\beta C) + (A\beta\gamma)$$

$$(AB) = (ABC) + (AB\gamma)$$

The student should write down similar relations for other frequencies of the classes of the first and the second order. In each case the right hand is the sum of all these ultimate frequencies which contain the letters representing the class frequency on the left.

**The set of ultimate frequencies is therefore called a fundamental set.** In general a set of frequencies is called a fundamental set if the frequency of every class can be expressed in terms of the frequencies of this set. In practice this means that if the frequencies of a fundamental set are known then the frequency of every other class can be calculated.

An important fundamental set is the set of so called positive frequencies along with  $N$ , the total frequency. This set consists of the following frequencies.  $\{(A), (B), (C), (AB), (AC), (BC), (ABC), N\}$

The presence of an attribute is termed a positive observation and its absence a negative one. Hence all the above frequencies, which are of those classes in which either one or more attributes are present are called positive frequencies. The corresponding classes are called positive classes. To express any other class frequency in terms of the frequencies of this set the method illustrated below is used.

To find  $(A\beta)$

$$A\beta = A(1 - B) = A - AB = (A) - (AB)$$

**The rule is:** replace  $\alpha, \beta, \gamma$  by  $(1-A), (1-B), (1-C)$  respectively and expand the symbol for the class as an algebraic expression. Then replace each symbol by the corresponding frequency. If the number 1 occurs in the expansion on the right, replace it by  $N$ .

Thus To find  $(A\beta\gamma)$

$$A\beta\gamma = A(1 - B)(1 - C) = A(1 - C - B + BC) = (A - AC - AB + ABC)$$

$$(A\beta\gamma) = (A) - (AC) - (AB) + (ABC)$$

## 5.2 Consistency of Data:-

Many times the data provides only the frequencies of a fundamental set. The frequencies of other classes have then to be obtained from these. Before doing this it is necessary to check the (arithmetical) consistency of the data. This check is based on the fact that the frequency of any class cannot be negative. Since the frequency of any class can be expressed as the sum of some ultimate frequencies, none of the class frequencies will be negative if the ultimate frequencies are not negative. Hence the condition for the consistency of data is that none of the ultimate frequencies should be negative. If the data provides the ultimate frequencies, this is easy to check. However, if the fundamental set provided by the data consists of the positive frequencies along with the total frequency further investigation is necessary.

**Condition for consistency of data:**

For the frequencies of classes of different orders, the conditions for consistency are as follows:

**Case (1) Single attribute A**

<b>A</b>	$\alpha$	<b>Total</b>
(A)	( $\alpha$ )	N

(i)  $(A) \geq 0$

(ii)  $(\alpha) \geq 0 \Rightarrow N - (A) \geq 0 \Rightarrow N \geq (A)$

**Case (2) Two attributes A and B**

	<b>A</b>	$\alpha$	<b>Total</b>
<b>B</b>	(AB)	( $\alpha B$ )	(B)
$\beta$	(A $\beta$ )	( $\alpha\beta$ )	( $\beta$ )
<b>Total</b>	(A)	( $\alpha$ )	N



- (i)  $(AB) \geq 0$   
(ii)  $(\alpha B) \geq 0 \Rightarrow B - (AB) \geq 0 \Rightarrow B \geq (AB)$   
(iii)  $(A\beta) \geq 0 \Rightarrow A - (AB) \geq 0 \Rightarrow A \geq (AB)$   
(iv)  $(\alpha\beta) \geq 0 \Rightarrow N - (A) - (B) + (AB) \geq 0 \Rightarrow (AB) \geq (A) + (B) - N$

**Case (3) Three attributes A, B and C**

	A			$\alpha$			Total		
	B	$\beta$	Total	B	$\beta$	Total	B	$\beta$	Total
C	(ABC)	(A $\beta$ C)	(AC)	( $\alpha$ BC)	( $\alpha\beta$ C)	( $\alpha$ C)	(BC)	( $\beta$ C)	(C)
$\gamma$	(A $B\gamma$ )	(A $\beta\gamma$ )	(A $\gamma$ )	( $\alpha$ B $\gamma$ )	( $\alpha\beta\gamma$ )	( $\alpha\gamma$ )	(B $\gamma$ )	( $\beta\gamma$ )	( $\gamma$ )
Total	(AB)	(A $\beta$ )	(A)	( $\alpha$ B)	( $\alpha\beta$ )	( $\alpha$ )	(B)	( $\beta$ )	N

- (i)  $(ABC) \geq 0$   
(ii)  $(A $B\gamma$ ) \geq 0 \Rightarrow (AB) - (ABC) \geq 0$   
(iii)  $(A\beta C) \geq 0 \Rightarrow (AC) - (ABC) \geq 0$   
(iv)  $(\alpha BC) \geq 0 \Rightarrow (BC) - (ABC) \geq 0$   
(v)  $(A\beta\gamma) \geq 0 \Rightarrow (A\beta) - (A\beta C) \geq 0$   
(vi)  $(\alpha B\gamma) \geq 0 \Rightarrow (\alpha B) - (\alpha BC) \geq 0$   
(vii)  $(\alpha\beta C) \geq 0 \Rightarrow (\alpha\beta) - (\alpha\beta C) \geq 0$   
(viii)  $(\alpha\beta\gamma) \geq 0 \Rightarrow (\alpha\beta) - (\alpha\beta C) \geq 0$   
(ix)  $(AB) + (BC) + (AC) \geq (A) + (B) + (C) - N$

**5.3 Methods to study Independence and Association of Attribute:-**

**Independence of Attributes (Proportion method):-**

If the attributes are said to be independent the presence or absence of the attribute does not affect the presence or absence of the other.

**For example:** The attributes skin colour and intelligence of persons are independent.

When the attribute A is not related with the attribute B, then the attributes A and B are said to be independent.

**OR**

Two attribute are said to be independent if there does not exist any kind of relationship.

Suppose A represents male and B represents successful person. Then we can say success and sex are independent if

$$\left( \begin{array}{l} \text{Proportion of success} \\ \text{among males} \end{array} \right) = \left( \begin{array}{l} \text{Proportion of success} \\ \text{among females} \end{array} \right)$$

$$\frac{(AB)}{(A)} = \frac{(\alpha B)}{(\alpha)}$$

By properties of ratio we can write

$$\frac{(AB)}{(A)} = \frac{(\alpha B)}{(\alpha)} = \frac{(AB) + (\alpha B)}{(A) + (\alpha)} = \frac{(B)}{N} \quad \left( \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} \right)$$

$$(AB) = \frac{(A) \times (B)}{N}$$

It means that attributes A and B are independent.

**Note:**

- [i] If A and B are independent then,  $\alpha$  and B are independent
- [ii] If A and B are independent then, A and  $\beta$  are independent
- [iii] If A and B are independent then,  $\alpha$  and  $\beta$  are independent
- [iv] If  $(AB)(\alpha\beta) = (A\beta)(\alpha B)$  then the attributes A and B are independent

**Association of Attributes (Comparison method):-**

Two attributes A and B are said to be associated if they are not independent. i. e. they are related in some way or the other. Hence the association can be statistically defined as following.

If attributes A and B are said to be positively associated if  $(AB) > \frac{(A) \times (B)}{N}$

If attributes A and B are said to be negatively associated if  $(AB) < \frac{(A) \times (B)}{N}$

If attributes A and B are not independent, then  $(AB) \neq \frac{(A) \times (B)}{N}$

If attributes A and B are independent, then  $(AB) = \frac{(A) \times (B)}{N}$

A and B are said to be completely associated if  $(AB) = (A)$  and  $(A\beta) = 0$   
 $(AB) = (B)$  and  $(\alpha B) = 0$

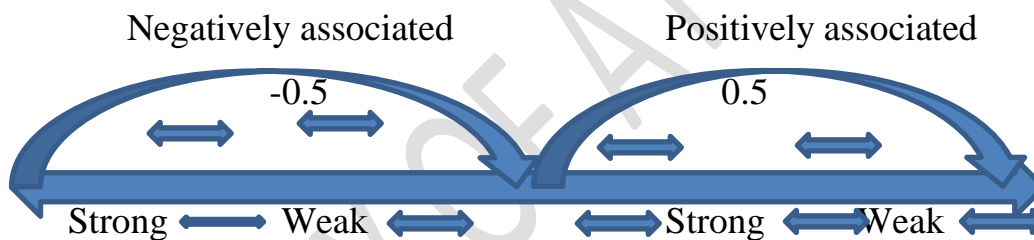
A and B are said to be completely dissociated if  $(AB) = 0$  or  $(\alpha\beta) = 0$

This method of detecting association is also termed as Frequency method

### Coefficient of Association:-

Above two methods gives rough idea about association but not the degree of association. Prof. George Udny Yule (1871-1951) has suggested a formula to measure of association and degree of association between two attributes A and B. It is denoted by Q and defined as follows.

### Yule's Coefficient of association



$$Q_{AB} = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

Where,

A and B are the presence of two attributes and  $\alpha, \beta$  are the absence of those two corresponding attributes.

### Note:

- [i] If  $(AB)$  or  $(\alpha\beta) = 0$  then attributes are completely dissociated.
- [ii] If  $(A\beta)$  or  $(\alpha B) = 0$  then attributes are completely associated.
- [iii] If  $-1 \leq Q_{AB} \leq -0.5$  then attributes are strong negatively associated.
- [iv] If  $-0.5 \leq Q_{AB} \leq 0$  then attributes are weak negatively associated.
- [v] If  $0 \leq Q_{AB} \leq 0.5$  then attributes are weak positively associated.

[vi] If  $0.5 \leq Q_{AB} \leq 1$  then attributes are strong positively associated.

[vii] If  $(AB)(\alpha\beta) \neq (A\beta)(\alpha B)$  then the attribute are associated or dissociated.

[viii]  $(AB)(\alpha\beta) - (A\beta)(\alpha B)$  it measures the amount of association.

**Examples:**

[1] From the given data find out the missing frequencies  $(AB) = 100$ ,  $(A) = 300$ ,  $(N) = 1000$ ,  $(B) = 600$

**Solution :-** Putting these values in the nine square table

	A	$\alpha$	Total
B	$(AB) = 100$	$(\alpha B) = 500$	$(B) = 600$
$\beta$	$(A\beta) = 200$	$(\alpha\beta) = 200$	$(\beta) = 400$
Total	$(A) = 300$	$(\alpha) = 700$	$N = 1000$

The missing frequencies are:  $(A\beta)$ ,  $(\alpha B)$ ,  $(\alpha\beta)$  and  $(\beta)$

[i]  $(A\beta) = (A) - (AB)$   
 $(A\beta) = 300 - 100$   
 $(A\beta) = 200$

[ii]  $(\alpha B) = (B) - (AB)$   
 $(\alpha B) = 600 - 100$   
 $(\alpha B) = 500$

[iii]  $(\alpha\beta) = (\beta) - (A\beta)$   
 $(\alpha\beta) = 400 - 200$   
 $(\alpha\beta) = 200$

[iv]  $(\beta) = (N) - (B)$   
 $(\beta) = 1000 - 600$   
 $(\beta) = 400$

Thus the missing frequencies are,  $(A\beta)=200$ ,  $(\alpha B)=500$ ,  $(\alpha\beta)=200$ ,  $(\alpha)=700$ ,  $(\beta)=400$ .

[2] From the following ultimate class frequencies find positive and negative classes and the total number of observations from a nine square table.

$(AB)=100$ ,  $(\alpha B)=80$ ,  $(A\beta)=40$

**Solution:-** Substituting the given values in the nine square table

	A	$\alpha$	Total
B	$(AB) = 100$	$(\alpha B) = 80$	$(B)=180$
$\beta$	$(A\beta) = 50$	$(\alpha\beta) = 40$	$(\beta)=90$
Total	$(A) = 150$	$(\alpha) = 120$	$N=270$

Frequencies of positive classes

[i]  $(A) = (AB) + (A\beta)$   
 $(A) = 100 + 50$   
 $(A) = 150$

[ii]  $(B) = (AB) + (\alpha B)$   
 $(B) = 100 + 80$   
 $(B) = 180$

Frequencies of negative classes

[iii]  $(\alpha) = (\alpha B) + (\alpha\beta)$   
 $(\alpha) = 80 + 40$   
 $(\alpha) = 120$

[iv]  $(\beta) = (A\beta) + (\alpha\beta)$   
 $(\beta) = 50 + 40$   
 $(\beta) = 90$

$$[v] \quad N = (AB) + (A\beta) + (\alpha B) + (\alpha\beta)$$

$$N = 100 + 50 + 80 + 40$$

$$N = 270$$

### Consistency of Data

[3] From the following two cases find out whether the data are consistent or not

Case 1: (A)=100; (B)=150; (AB)=60;(N)=500

Case 2: (A)=100; (B)=150; (AB)=140;(N)=500

#### Solution:

Case 1: We are given: (A)=100; (B)=150; (AB)=60;(N)=500

Substitute these values in the nine-square table

	A	$\alpha$	Total
B	(AB) = 60	( $\alpha B$ ) = 90	(B)=150
$\beta$	(A $\beta$ ) = 40	( $\alpha\beta$ ) = 310	( $\beta$ )=350
Total	(A) =100	( $\alpha$ ) = 400	N=500

From the table, we can write

$$(A\beta) = (A) - (AB) = 100 - 60 = 40$$

$$(\alpha B) = (B) - (AB) = 150 - 60 = 90$$

$$(\alpha\beta) = (\alpha) - (\alpha B) = 400 - 90 = 310$$

**Interpretation:-** Since all the ultimate classes are positive, we conclude that the given data are consistent.

#### Case-2:-

Given values are: (A)=100; (B)=150; (AB)=140;(N)=500

By putting this values in the nine-square table

We can determine the missing values.

	A	$\alpha$	Total
B	(AB) = 140	( $\alpha B$ ) = 10	(B)=150

$\beta$	$(A\beta) = -40$	$(\alpha\beta) = 390$	$(\beta) = 350$
Total	$(A) = 100$	$(\alpha) = 400$	$N = 500$

From the table, we can write

$$(A\beta) = (A) - (AB) = 100 - 140 = -40$$

$$(\alpha B) = (B) - (AB) = 150 - 140 = 10$$

$$(\alpha\beta) = (\alpha) - (\alpha B) = 400 - 10 = 390$$

**Interpretation:-** Since one of the ultimate class frequencies, i.e.,  $(A\beta)$  is negative and hence the given data are inconsistent.

[4] In a group of 200 persons, 142 read Marathi newspaper, 108 read English newspaper and 85 read both the papers. Find the number of persons reading (i) only Marathi newspaper (ii) newspaper in only one language (iii) no newspaper

**Solution:-** Let A denotes read Marathi newspaper and B denotes read English newspaper

	A	$\alpha$	Total
B	$(AB) = 85$	$(\alpha B) = 23$	$(B) = 108$
$\beta$	$(A\beta) = 57$	$(\alpha\beta) = 35$	$(\beta) = 92$
Total	$(A) = 142$	$(\alpha) = 58$	$N = 200$

(i) Read only Marathi newspaper is  $(A\beta) = 57$

(ii) Read newspaper in only one language is  $(A\beta) + (\alpha B) = 57 + 23 = 80$

(iii) Read no newspaper is  $(\alpha\beta) = 35$

[5] Out of 110 students interview for their liking in music 48 students liked Indian music. 73 liked Western music and 20 liked both. Find the number of students who did not like music.

**Solution:-** Let A denotes students like Indian music and B denotes students

like Western music.

	A	$\alpha$	Total
B	$(AB) = 20$	$(\alpha B) = 53$	$(B)=73$
$\beta$	$(A\beta) = 28$	$(\alpha\beta)= 9$	$(\beta)=37$
Total	$(A) =73$	$(\alpha) = 37$	$N=110$

Number of students who did not like music is  $(\alpha\beta)= 9$

[6] 500 students appeared for an examination of whom 275 were boys. Out of 350 successful students, 150 were boys. Find the number of (i) successful girls (ii) unsuccessful girls (iii) unsuccessful boys

**Solution:-** Let A denotes student is a boy and B denotes successful student.

	A	$\alpha$	Total
B	$(AB) = 150$	$(\alpha B) = 200$	$(B)=350$
$\beta$	$(A\beta) = 125$	$(\alpha\beta)=25$	$(\beta)=150$
Total	$(A) =275$	$(\alpha) = 225$	$N=500$

(i) Successful girls are  $(\alpha B) = 200$

(ii) Unsuccessful girls are  $(\alpha\beta)=25$

(iii) Unsuccessful boys are  $(A\beta) = 125$

[7] In a group of 100 employees in a firm, there were 80 males. The number of married employees was 60 among whom 30 were males. Examine whether the information is correct

**Solution:-** Let A denotes male employee and B denotes married employee.

	A	$\alpha$	Total
B	$(AB) = 30$	$(\alpha B) = 30$	$(B)=60$



$\beta$	$(A\beta) = 50$	$(\alpha\beta) = -10$	$(\beta) = 40$
Total	$(A) = 80$	$(\alpha) = 20$	$N = 100$

**Comment:**

Since,  $(\alpha\beta) = -10$  i.e.  $(\alpha\beta) < 0$ , hence, given information is incorrect.

[8] The data below gives the information of hobbies of a group of 100 girls in a school. 20 girls liked singing, 35 enjoyed painting and 12 girls favoured both hobbies. Are the data consistent?

**Solution:-** Let A denotes girl like singing and B denotes girl like painting

	A	$\alpha$	Total
B	$(AB) = 12$	$(\alpha B) = 23$	$(B) = 35$
$\beta$	$(A\beta) = 8$	$(\alpha\beta) = 57$	$(\beta) = 65$
Total	$(A) = 20$	$(\alpha) = 80$	$N = 100$

**Comment:**

Since, all frequencies are positive hence, given information is correct.

**Proportion method**

[9] Use proportion method to determine nature association between A and B:

	A	$\alpha$	Total
B	$(AB) = 30$	$(\alpha B) = 20$	$(B) = 50$
$\beta$	$(A\beta) = 50$	$(\alpha\beta) = 100$	$(\beta) = 150$
Total	$(A) = 80$	$(\alpha) = 120$	$N = 200$

**Solution:-** According to the proportion method if the attributes A and B are

independent i.e.  $\frac{(AB)}{(A)} = \frac{(\alpha B)}{(\alpha)}$

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We have,

$$(AB) = 30, (A) = 80, (\alpha B) = 20, (\alpha) = 120$$

$$\frac{(AB)}{(A)} = \frac{30}{80} = 0.375 \text{ and } \frac{(\alpha B)}{(\alpha)} = \frac{20}{120} = 0.167$$

Since,

$$\frac{(AB)}{(A)} > \frac{(\alpha B)}{(\alpha)}$$

**Interpretation:** The attribute A and B are positively associated.

[10] In a population of 500 students the number of married is 200 out of 150 students who failed 60 belonged to the married group. It is required to find out whether the attributes marriage and failure are independent. Positively associated or negatively associated.

**Solution:-**

Let A denote married student then  $\alpha$  denote unmarried student.

B denote number of failures then  $\beta$  denote non failure

We are given the total number of students

i.e.,  $N=500$ ;  $(A) = 200$ ;  $(B) = 150$  &  $(AB)$

i.e., the number of married students who failed,  $= 60$

According to the proportion method if the attributes A and B are independent

$$\text{i.e. } \frac{(AB)}{(A)} = \frac{(\alpha B)}{(\alpha)}$$

In other words, if the proportion of married students who failed is the same as the proportion of unmarried student who failed say that the attribute marriage and failure are independent

Proportion of unmarried student who failed:

$$\text{i. e. } \frac{(AB)}{(A)} = \frac{60}{200} = 0.3 \text{ or } 30\%$$

Proportion of unmarried students who failed:

$$\frac{(\alpha B)}{(\alpha)} = \frac{90}{300} = 0.3 \text{ or } 30\%$$

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**Interpretation:** Since the proportions are the same we conclude that the attributes, marriage and failure, are independent

[11] Out of 3000 unskilled workers of a factory, 2000 come from rural areas and out of 1200 skilled workers, 300 come from rural areas. Determine the association between skill and residence by the method of proportions

**Answer:-** Let A denote skilled workers

$\therefore \alpha$  will denote unskilled workers

Let B denote workers from rural areas

$\therefore \beta$  will denote workers from urban areas

The given information is:

(A)=1200; ( $\alpha$ )=3000 ; (B)=2000, ( $\alpha\beta$ ) =2000; (AB)=300

According to the method of Proportions, two attributes A & B are said to be independent if :

$$\frac{(AB)}{(A)} = \frac{(\alpha\beta)}{(\alpha)}$$

In the given case:  $\frac{(AB)}{(A)} = \frac{300}{1200} = 0.25$  and  $\frac{(\alpha\beta)}{(\alpha)} = \frac{2000}{3000} = 0.64$

**Interpretation:** Since,  $\frac{(AB)}{(A)}$  is less than  $\frac{(\alpha\beta)}{(\alpha)}$ , there is negative association between skill and residence.

[12] Comment on the following statement:

(a) “99% of the people who drink beer, die before reaching 100 year of age therefore, drinking beer is bad for longevity”.

(b) “road accidents resulted in 5012 deaths in 1988 in India and 8613 in 1989 while the number of women make bad drivers”

**Answer:** (a) we are not given complete information. i.e. what percentage of people who do not drink beer die before reaching 100 years of age and as such the inference drawn above is wrong . It is possible that 100% of the persons who do not drink beer may die before reaching 100 year of age in

which case drinking may be found to be good for longevity. Therefore, for association between A & B in addition to  $\frac{(AB)}{(A)}$ . It is necessary to know  $\frac{(\alpha\beta)}{(\alpha)}$  also.

(b) on the basis of the information given it cannot be concluded that the women make bad drivers . women can be regarded as bad drivers only if number of accidents made per woman driver is more than be the number of accidents made by a man driver . These figures are not given. The increase in the number accidents from 5082 in 1988 to 8623 in 1990 may be not due to bad driving but due to other factors like increase in population, increase in number of vehicles on the road , more congested roads , etc. hence the statement is an example of illusory association .

[13] In two towns A & B, the following information was supplied by an investigator:

	Town A	Town B
Total population (in thousands)	240	234
Literates	40	34
Illiterate criminals	50	20
Literate criminals	5	2

Compare the degree of association between literacy and crime in each of the two towns.

**Answer:** Let A denote literates

$\therefore \alpha$  will denote illiterates

Let B denote criminals

$\therefore \beta$  will denote non – criminals

In terms of these symbols the given information is expressed as below:

	Town A	Town B
N	240	234
(A)	40	34
( $\alpha$ B)	50	20
(AB)	5	2

The missing frequencies can be ascertained by the nine-square table

Town A

	A	$\alpha$	Total
B	(AB) = 5	( $\alpha$ B) = 50	(B) = 55
$\beta$	(A $\beta$ ) = 35	( $\alpha\beta$ ) = 150	( $\beta$ ) = 185
Total	(A) = 40	( $\alpha$ ) = 200	N = 240

Town B

	A	$\alpha$	Total
B	(AB) = 2	( $\alpha$ B) = 20	(B) = 22
$\beta$	(A $\beta$ ) = 32	( $\alpha\beta$ ) = 180	( $\beta$ ) = 212
Total	(A) = 34	( $\alpha$ ) = 200	N = 234

Thus, there is dissociation between literacy and criminality in both towns, the degree of dissociation is slightly more in town B compared to town A.

[14] The male population of U.P. is 250 lakhs. The number of literate males is 20 lakhs and the total number of criminals is 26 thousand. The number of literate male criminals is 2 thousand. Do you find any association between literacy and criminality?

**Answer:-** Let A denote literate males

$\therefore \alpha$  will denote illiterate males

Let B denote male criminals

$\therefore \beta$  will denote male non criminals

The given information is in lakhs

N=250; (A)=20; (B)=0.26; (AB)=0.02

---

$$\text{Expectation of } (AB) = \frac{(A) \times (B)}{N} = 0.0208$$

**Interpretation:-** Since (AB) actual observation (0.02) is less than expectation (0.0208), the attributes literacy and criminality are negatively associated, i.e. literacy checks criminality.

### Independent method

Examine for attributes A and B are Independent / Positively associated / Negatively associated for the following:

$$[15] N = 100, (A) = 60, (B) = 50, (AB) = 30$$

We have,

$$(AB) = \frac{(A) \times (B)}{N}$$

$$\frac{60 \times 50}{100} = 30$$

But, given (AB) = 30

$$\Rightarrow (AB) = \frac{(A) \times (B)}{N}$$

Therefore, A and B are Independent

$$[16] N = 200, (A) = 80, (B) = 50, (AB) = 25$$

We have,

$$(AB) = \frac{(A) \times (B)}{N}$$

$$\frac{80 \times 50}{200} = 20$$

But, given (AB) = 25

$$\Rightarrow (AB) > \frac{(A) \times (B)}{N}$$

Therefore, A and B are positively associated

$$[17] N = 500, (A) = 325, (B) = 310, (AB) = 160$$

We have,

$$(AB) = \frac{(A) \times (B)}{N}$$

$$\frac{325 \times 310}{500} = 201.5$$

But, given  $(AB) = 160$

$$\Rightarrow (AB) < \frac{(A) \times (B)}{N}$$

Therefore, A and B are negatively associated

$$[18] (AB) = 128; (\alpha B) = 384; (A\beta) = 24; (\alpha\beta) = 72$$

First, we calculate

$$(AB) \times (\alpha\beta) = 128 \times 72 = 9216 \dots\dots(i)$$

$$(A\beta) \times (\alpha B) = 384 \times 24 = 9216 \dots\dots(ii)$$

From (i) and (ii) we can write

$$(AB) \times (\alpha\beta) = (A\beta) \times (\alpha B)$$

Therefore, we conclude that attribute A and B are independent

$$[19] (AB) = 30; (\alpha B) = 120; (A\beta) = 90; (\alpha\beta) = 360$$

First, we calculate

$$(AB) \times (\alpha\beta) = 30 \times 360 = 10800 \dots\dots(i)$$

$$(A\beta) \times (\alpha B) = 90 \times 120 = 10800 \dots\dots(ii)$$

From (i) and (ii) we can write

$$(AB) \times (\alpha\beta) = (A\beta) \times (\alpha B)$$

Therefore, we can conclude that attribute A and B are independent

	A	$\alpha$	Total
B	$(AB) = 40$	$(\alpha B) = 200$	$(B) = 240$
$\beta$	$(A\beta) = 60$	$(\alpha\beta) = 100$	$(\beta) = 160$
Total	$(A) = 100$	$(\alpha) = 300$	$N=400$

**Solution:-** Attribute A and B shall be called independent if  $(AB) = \frac{(A) \times (B)}{N}$

$$\begin{aligned} \text{Expectation of } (AB) &= \frac{100 \times 240}{400} \\ &= \frac{24000}{400} \\ &= 60 \end{aligned}$$

Since (AB) actual Observation (40) is less than the expectation (60)

i. e. (AB) < Expectation of (AB)

**Interpretation:** The attributes marriage and success in the examination are negatively associated

[20] Find A and B are independent positively associated or negatively associated from the data given below: (A)=470, (B)=620, (AB)=320, N=1000

**Solution:-** Attribute A and B shall be called independent if

$$\begin{aligned} (AB) &= \frac{(A) \times (B)}{N} \\ (AB) &= 320, (A) = 470, (B) = 620, N = 1000 \\ \text{Expectation of } (AB) &= \frac{470 \times 620}{1000} \\ &= \frac{291400}{1000} \\ &= 291.4 \end{aligned}$$

Since (AB) actual observation (320) is more than the expectation (291.4)

i. e. (AB) > Expectation of (AB)

**Interpretation:** Attributes A and B are positively associated

#### Yule's coefficient of association method:-

[21] Every-eight resident of an Indian city, who were interviewed during a sample survey, are classified below according to their smoking and tea drinking habits. Calculate Yule's coefficient of association and comment on its values.

	smokers	Non -smokers
Tea-drinkers	40	33



Non-tea-drinkers	3	12
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**Solution:-** Let A denote smokers

∴ α would denote non –smokers

Let B denote tea drinkers

∴ β would denote non-tea drinkers

The given data in terms of these symbols are

(AB), i.e. no. of smokers and tea drinkers = 40

(Aβ), i.e. no. of smokers and non –tea-drinkers = 3

(αB), i.e. no. of tea drinkers and non-smokers = 33

(αβ), i.e. no. of non-smokers and non-tea drinkers = 12

Applying Yule’s method ;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

Substituting the values of (AB) , (Aβ), (αB) and (αβ) in this formula

$$= \frac{480-99}{480+99}$$

$$= \frac{381}{579}$$

$$Q = 0.658$$

**Interpretation:** This shows that the attribute tea drinking and smoking are positively associated.

[22] A teacher examined 280 students in Economics and Auditing and found that 160 failed in Economics, 140 failed in Auditing and 80 failed in both the subjects, is there any association between failure in Economics and Auditing ?

**Solution:-**Let A denote students who failed in Economics and B denote student who failed in Auditing

Putting the given information in a nine- square table, we have

	A	$\alpha$	Total
B	$(AB) = 80$	$(\alpha B) = 60$	$(B) = 140$
$\beta$	$(A\beta) = 80$	$(\alpha\beta) = 60$	$(\beta) = 140$
Total	$(A) = 160$	$(\alpha) = 120$	$N=280$

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(80 \times 60) - (80 \times 60)}{(80 \times 60) + (80 \times 60)}$$

$$Q = \frac{4800 - 4800}{4800 + 4800}$$

$$Q = 0$$

**Interpretation:-** Since, Yules coefficient of association is zero and therefore there is no association between failure in Economic and Auditing .

[23] In an examination at which 500 candidates appeared boys out numbered girls by 40 percent of all candidates. Number of passed candidates exceeded the number of failed candidates by 300. Boys failing in examination numbered 80. Construct the 9 square table and calculate the coefficient of association between boys and success in the examination

**Solution:-** Let A denotes boys and  $\alpha$  will denotes girls

Let B denote those who passed and  $\beta$  will denote those who failed

From the given information

$$N=500, (AB) = 80$$

Let X denote girls, boys would be  $X + 70$  (14% of 500)

$$X + X + 70 = 500 \text{ [boys + girls = total]}$$

$$2X = 500 - 70$$

$$2X = 430$$

$$X = \frac{430}{2}$$

$$X = 215$$

OR

Hence, girls ( $\alpha$ ) = 215 and boys (A) = 500 - 215 = 285

Let X denote failed candidates therefore passed candidates = 300 + X

Total candidates = 500

$$\text{Hence, } X + 300 + X = 500$$

$$2X = 500 - 300$$

$$2X = 200$$

$$X = 100$$

Substitute the given information in a nine square table

	A	$\alpha$	Total
B	(AB) = 205	( $\alpha$ B) = 195	(B) = 400
$\beta$	(A $\beta$ ) = 80	( $\alpha\beta$ ) = 20	( $\beta$ ) = 100
Total	(A) = 285	( $\alpha$ ) = 215	N = 500

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(250 \times 20) - (80 \times 195)}{(250 \times 20) + (80 \times 195)}$$

$$Q = \frac{4100 - 15600}{4100 + 15600}$$

$$Q = \frac{11500}{19700}$$

$$Q = -0.584$$

**Interpretation:-**Since, Yules coefficient of association is -0.584 and therefore there is negative association between boys and success.

[24] Prepare  $2 \times 2$  table from the following information. Calculate Yule's coefficient off association and interpret the result:

$N=1500$  ;  $(\alpha)=1117$ ;  $(B)=360$ ;  $(AB)=35$

**Solution:-**by putting the known values in the nine-square table, we can find out the unknown values

	A	$\alpha$	Total
B	$(AB) = 35$	$(\alpha B) = 325$	$(B) = 360$
$\beta$	$(A\beta) = 348$	$(\alpha\beta) = 792$	$(\beta) = 1140$
Total	$(A) = 383$	$(\alpha) = 1117$	$N=1500$

Thus, Yule's coefficient of association

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(35 \times 792) - (348 \times 325)}{(35 \times 792) + (348 \times 325)}$$

$$Q = \frac{27720 - 113100}{27720 + 113100}$$

$$Q = \frac{-85380}{140820} = -0.606$$

**Interpretation:-**Since, Yules coefficient of association is negative and therefore attributes are negatively associated.

[25] In a co-education Institute , out of 200 students 150 were boys. They took an examination and it was found that 120 passed, 10 girls had failed, is there any association between sex and success in the examination.

**Solution:-**Let A denotes boys  $\therefore \alpha$  will denotes girls  
B denotes passed  $\therefore \beta$  will denotes those who failed.

We are given :  $N=200$ ;  $(A)=150$ ;  $(AB)=120$ ;  $(\alpha\beta)=10$ .

Other frequencies can be obtained from the nine-square table:

	A	$\alpha$	Total
B	$(AB) = 120$	$(\alpha B) = 40$	$(B) = 160$
$\beta$	$(A\beta) = 30$	$(\alpha\beta) = 10$	$(\beta) = 40$
Total	$(A) = 150$	$(\alpha) = 50$	$N=200$

Applying Yule's method

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(120 \times 10) - (30 \times 40)}{(120 \times 10) + (30 \times 40)}$$

$$Q = \frac{1200 - 1200}{1200 + 1200}$$

$$Q = 0$$

**Interpretation:-**Since, Yules coefficient of association is zero, hence there is no association between sex and success in the examination

[26] In a class test in which 135 candidate were examined for proficiency in English & Economics, it was discovered that 75 students failed in English, 80 failed in Economics, and 50 failed in both. Find if there is any association between failing in English and Economics and also state the magnitude of association.

**Solution:-**Let A denote those who failed in English

$\therefore \alpha$  will denote those who passed in English

Let B denote failed in Economics

$\therefore \beta$  will denote those passed in Economics.

Hence, given information can be put in a nine square table and their values determined.

	A	$\alpha$	Total
B	$(AB) = 50$	$(\alpha B) = 40$	$(B) = 90$

$\beta$	$(A\beta) = 25$	$(\alpha\beta) = 20$	$(\beta) = 45$
Total	$(A) = 75$	$(\alpha) = 60$	$N=135$

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(50 \times 20) - (25 \times 40)}{(50 \times 20) + (25 \times 40)}$$

$$Q = \frac{(1000) - (1000)}{(1000) + (1000)}$$

$$Q = 0$$

**Interpretation:-**Since, Yules coefficient of association is zero there is no association between failure in English and Economic.

[27] From the following data, Find the association between darkness of eye-colour in father and son;

Father with dark eyes and sons with dark eyes is 140

Father with dark eyes and sons with not dark eyes is 160

Father with not dark eyes and sons with dark eyes is 180

Father with not dark eyes and sons with not dark eyes is 120

What would have been the frequency of father with dark eyes and sons with dark eyes for the same total number had there been complete independence?

**Solution:-**Let A denote father with dark eyes

$\therefore \alpha$  would denote fathers with not dark eyes the given data in terms of these symbols are,  $(AB) = 1040$ ;  $(A\beta) = 160$ ;  $(\alpha B) = 180$ ;  $(\alpha\beta) = 120$

	A	$\alpha$	Total
B	$(AB) = 1040$	$(\alpha B) = 180$	$(B) = 1220$
$\beta$	$(A\beta) = 160$	$(\alpha\beta) = 120$	$(\beta) = 280$
Total	$(A) = 1200$	$(\alpha) = 300$	$N=1500$

Applying Yule's method ;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

Substituting the values of (AB), (Aβ), (αB) and (αβ) in this formula

$$Q = \frac{(1040 \times 120) - (160 \times 180)}{(1040 \times 120) + (160 \times 180)}$$

$$Q = \frac{(124800) - (28800)}{(124800) + (28800)}$$

$$Q = \frac{(96000)}{(153600)}$$

$$Q = 0.625$$

For finding out the frequency of fathers with dark eyes and sons with dark eyes for the same total number had there been complete independence we shall find out expectation of (AB)

$$\text{Expectation of (AB)} = \frac{(A) \times (B)}{N}$$

For calculating (A), (B) & N we will put the given information in nine-square table

	A	α	Total
B	(AB) = 1040	(αB) = 180	(B) = 1220
β	(Aβ) = 160	(αβ) = 120	(β) = 280
Total	(A) = 1200	(α) = 300	N = 1500

$$A=1200; B=1220; N=1500, \text{ hence expectation of (AB)} = \frac{(1200) \times (1220)}{1500} = 976$$

[28] Investigate the association between eye colour of wives from the data given below:

Husbands with light eyes and wives with light eyes = 309

Husbands with light eyes and wives with non- light eyes = 214

Husbands with non-light eyes & wives with light eyes = 132

Husbands with non-light eyes & wives with non-light eyes = 119

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**Solution:**

Since we have to find out the association between eye color of husband and that of wife, one attribute we would take As A and another as B.

Let A denote husbands with light eyes.

∴  $\alpha$  denote husband with non-light eyes.

Let B denote wives with light eyes

∴  $\beta$  denote wives with non-light eyes.

The given data in terms of symbols are

$$(AB)=309, (A\beta)=214, (\alpha A)=132, (\alpha\beta)=119$$

Applying Yule s Method:

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha A)}{(AB)(\alpha\beta) + (A\beta)(\alpha A)}$$

Substituting the above values in the formula

$$Q = \frac{(309 \times 119) - (214 \times 132)}{(309 \times 119) + (214 \times 132)} = \frac{8523}{65019} \\ = 0.131$$

**Interpretation:-** Thus, there is a very little association between the eye color of husband and wife

[29] Find the association between Literacy and unemployment from the following figures. Also, Comment on the results.

Total Adults	10000
Literates	1290
Literates Unemployed	820
Unemployed	1390

**Solution:**

Let A denote Literates

∴  $\alpha$  will denote illiterates

Let B denote Unemployed



∴ β denote employed

We are given;(A)=1290, (B)=1390 , (AB)=820 , N=10000. Putting this information in the nine square table and finding missing values.

	A	α	Total
B	(AB) = 820	(αB) = 570	(B) =1390
β	(Aβ) = 470	(αβ)= 8140	(β) = 8610
Total	(A) = 1290	(α) = 8710	N=10000

Applying Yule's method ;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(820 \times 8140) - (570 \times 470)}{(40 \times 12) + (3 \times 33)}$$

$$Q = \frac{(6674800) - (267900)}{(6674800) + (267900)}$$

$$Q = \frac{(6406900)}{(6942700)}$$

$$Q = 0.922$$

**Interpretation:-**There is a high degree of positive association between literacy and unemployment.

[30] In an assorted study to find whether tall husband tend to marry tall wives, the following information about the wives of 1250 tall and 1250 short statured husband was published find the coefficient of association between the stature of the wives and husband.

	Tall husbands	short husbands
Tall wives	60%	10%
Short wives	10%	50%

**Solution:-**Let A denotes tall husband and α denotes short husbands

Let B will denotes tall wives and β denotes short wives

$$(AB) = 60, (A\beta) = 10, (\alpha B) = 10, (\alpha\beta) = 50$$

---

Applying Yule's method ;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(60 \times 50) - (10 \times 10)}{(60 \times 50) + (10 \times 10)}$$

$$Q = \frac{(300) - (100)}{(300) + (100)}$$

$$Q = \frac{(200)}{(400)}$$

$$Q = 0.5$$

**Interpretation:-** There is a positive association between tall husband and tall wives.

[31] Do you find any association between the temperaments of brother and sisters from the following data :

Good natured brother and good natured sisters	1230
Good natured brother and sullen sisters	850
Sullen brother and good sisters	530
Sullen brother and sullen sisters	980

**Solution:-** Let A denotes good natured brother

B denotes good natured sisters

$\alpha$  denotes sullen brothers and  $\beta$  denotes sullen sisters

We are given that  $(AB) = 1230$ ,  $(A\beta) = 850$ ,  $(\alpha B) = 530$ ,  $(\alpha\beta) = 980$

Applying Yule's method ;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{(1230 \times 980) - (850 \times 530)}{(1230 \times 980) + (850 \times 530)}$$

$$Q = \frac{(1205400) - (450500)}{(1205400) + (450500)}$$

$$Q = \frac{(754900)}{(1655900)}$$

$$Q = 0.46$$

**Interpretation:-** There is a positive association between good natured brother and good natured sister.

[32] Among 252 candidates, 140 were boys, 72 candidates were successful, among them 40 were boys. Obtain coefficient of association and comment on result

**Solution:-**

Let A denotes candidate is a boy and B denotes successful candidate

$\alpha$  denotes sullen brothers and  $\beta$  denotes sullen sisters

	A	$\alpha$	Total
B	(AB) = 40	( $\alpha$ B) = 32	(B) = 72
$\beta$	(A $\beta$ ) = 100	( $\alpha\beta$ ) = 80	( $\beta$ ) = 180
Total	(A) = 140	( $\alpha$ ) = 112	N=252

Applying Yule's method ;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{40 \times 80 - 100 \times 32}{40 \times 80 + 100 \times 32}$$

$$Q = \frac{3200 - 3200}{3200 + 3200}$$

$$Q = \frac{0}{6400}$$

$$Q = 0$$

**Interpretation:-**Since, Yules coefficient of association is zero there is no association between sex and success.

[33] In an institute out of 200 students 150 were boys. An examination 120 boys and 40 girls passed. Find Yule's coefficient of association between sex and success and comment on the result:

**Solution:-**Let A denotes student is a boy and B denotes student passed the examination

And  $\alpha$  denotes student is a girl and  $\beta$  denotes student failed the examination

	A	$\alpha$	Total
B	(AB) = 120	( $\alpha$ B) = 40	(B) = 160
$\beta$	(A $\beta$ ) = 30	( $\alpha\beta$ ) = 10	( $\beta$ ) = 40
Total	(A) = 150	( $\alpha$ ) = 50	N=200

Applying Yule's method;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{120 \times 10 - 30 \times 40}{120 \times 10 + 30 \times 40}$$

$$Q = \frac{1200 - 1200}{1200 + 1200}$$

$$Q = \frac{0}{2400}$$

$$Q = 0$$

**Interpretation:-**Since, Yules coefficient of association is zero and therefore there is no association between sex and success.

[34] Out of 200 students that appeared for M.B.A. examination, 80 were married. Among 60 students who failed, 24 were married. Find the coefficient of association between marriage and failure in the examination. Comment on your result:

**Solution:-**Let A denotes student is married and B denotes student failed the examination and  $\alpha$  denotes student is unmarried and  $\beta$  denotes student passed the examination

	A	$\alpha$	Total
B	(AB) = 24	( $\alpha$ B) = 36	(B) = 60
$\beta$	(A $\beta$ ) = 56	( $\alpha\beta$ ) = 84	( $\beta$ ) = 140
Total	(A) = 80	( $\alpha$ ) = 120	N=200

Applying Yule's method;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{24 \times 84 - 56 \times 36}{24 \times 84 + 56 \times 36}$$

$$Q = \frac{2016 - 2016}{2016 + 2016}$$

$$Q = \frac{0}{4032}$$

$$Q = 0$$

**Interpretation:-**Since, Yule's coefficient of association is zero and therefore there is no association between marriage and failure.

[35] 400 students, 160 were married. Among 120 students who failed, 48 were married. Find coefficient of association between attributes marriage and failure.

**Solution:-**Let A denotes student is married and B denotes student failed the examination and  $\alpha$  denotes student is unmarried and  $\beta$  denotes student passed the examination

	A	$\alpha$	Total
B	(AB) = 48	( $\alpha$ B) = 72	(B) = 120
$\beta$	(A $\beta$ ) = 112	( $\alpha\beta$ ) = 168	( $\beta$ ) = 280

Total	(A) = 160	( $\alpha$ ) = 240	N=400
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Applying Yule's method;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{48 \times 168 - 112 \times 72}{48 \times 168 + 112 \times 72}$$

$$Q = \frac{8064 - 8064}{8064 + 8064}$$

$$Q = \frac{0}{16128}$$

$$Q = 0$$

**Interpretation:-**Since, Yules coefficient of association is zero and therefore there is no association between marriage and failure.

[36] A & B represent attributes ' going for morning walk and physically fit'.

Compute Yule's coefficient

N = 200 ; (A) = 120 ; (B) = 100 ; (AB) = 80

	A	$\alpha$	Total
B	(AB) = 80	( $\alpha B$ ) = 20	(B) = 100
$\beta$	(A $\beta$ ) = 40	( $\alpha\beta$ ) = 60	( $\beta$ ) = 100
Total	(A) = 120	( $\alpha$ ) = 80	N= 200

Applying Yule's method;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{80 \times 60 - 40 \times 20}{80 \times 60 + 40 \times 20}$$

$$Q = \frac{4800 - 800}{4800 + 800}$$

$$Q = \frac{4000}{5600}$$

$$Q = 0.71$$

**Interpretation:-** There is positive association between two attributes.

[37] Calculate Yule's coefficient of association between the weight of the children and their respective economic condition.

	Poor children	Rich children
Below normal weight	<b>75</b>	<b>23</b>
Above normal weight	<b>5</b>	<b>42</b>

**Solution:-** Let A denotes child is below normal weight and B denotes child is poor and  $\alpha$  denotes child is above normal weight and  $\beta$  denotes child is rich

	A	$\alpha$	Total
B	$(AB) = 75$	$(\alpha B) = 5$	$(B) = 80$
$\beta$	$(A\beta) = 23$	$(\alpha\beta) = 42$	$(\beta) = 65$
Total	$(A) = 98$	$(\alpha) = 47$	$N = 145$

Applying Yule's method;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{75 \times 42 - 23 \times 5}{75 \times 42 + 23 \times 5}$$

$$Q = \frac{3150 - 115}{3150 + 3150}$$

$$Q = \frac{3035}{3265}$$

$$Q = 0.93$$

**Interpretation:-** There is high positive association between weight of the children and their respective economic condition.

[38] Calculate Yule's coefficient of association between the inoculation and not attacked by hepatitis

	Not attacked	Attacked
Inoculated	<b>431</b>	<b>5</b>
Not inoculated	<b>291</b>	<b>9</b>

**Solution:-** Let A denotes child is inoculated and B denotes not attacked by hepatitis and  $\alpha$  denotes child is not inoculated and  $\beta$  denotes attacked by hepatitis

	A	$\alpha$	Total
B	(AB) = 431	( $\alpha B$ ) = 291	(B) = 722
$\beta$	(A $\beta$ ) = 5	( $\alpha\beta$ ) = 9	( $\beta$ ) = 14
Total	(A) = 436	( $\alpha$ ) = 300	N = 736

Applying Yule's method;



$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{431 \times 9 - 5 \times 291}{431 \times 9 + 5 \times 291}$$

$$Q = \frac{3879 - 1455}{3879 + 1455}$$

$$Q = \frac{2424}{5334}$$

$$Q = 0.45$$

**Interpretation:-** There is positive association between inoculation and not attacked by hepatitis. Hence, we can say inoculation was effective for controlling hepatitis.

[39] 400 saplings were planted, 210 got flowered. Fertilizers were applied to 100 of which 85 got flowered. Discuss whether application of fertilizers is useful in flowering of plants

**Solution:-** Let A denotes sapling got flowered and B denotes fertilizer applied to sapling and  $\alpha$  denotes sapling not flowered and  $\beta$  denotes not applied fertilizer to sapling

	A	$\alpha$	Total
B	(AB) = 85	( $\alpha$ B) = 15	(B) = 100
$\beta$	(A $\beta$ ) = 125	( $\alpha\beta$ ) = 175	( $\beta$ ) = 300
Total	(A) = 210	( $\alpha$ ) = 190	N=400

Applying Yule's method;

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$Q = \frac{85 \times 175 - 125 \times 15}{85 \times 175 + 125 \times 15}$$

$$Q = \frac{14875 - 1875}{14875 + 1875}$$

$$Q = \frac{13000}{16750}$$

$$Q = 0.77$$

**Interpretation:-** There is positive association between got flowered and application of fertilizer. Hence, we can say application of fertilizers is useful in flowering of plants

**Problems on three attribute:-**

[40] Fill the following table:

$$(ABC) = 26; (AB\gamma) = 38; (A\beta C) = 18; (\alpha BC) = 12; (A\beta\gamma) = 12; (\alpha B\gamma) = 16;$$

$$(\alpha\beta C) = 6; (\alpha\beta\gamma) = 9$$

	A			$\alpha$			Total		
	B	$\beta$	Total	B	$\beta$	Total	B	$\beta$	Total
C	(AB) = 26	(A $\beta$ C) = 18	(AC) = 44	( $\alpha$ BC) = 12	( $\alpha\beta$ C) = 6	( $\alpha$ C) = 18	(BC) = 38	( $\beta$ C) = 24	(C) = 62
$\gamma$	(A $B\gamma$ ) = 38	(A $\beta\gamma$ ) = 12	(A $\gamma$ ) = 50	( $\alpha$ B $\gamma$ ) = 16	( $\alpha\beta\gamma$ ) = 9	( $\alpha\gamma$ ) = 25	(B $\gamma$ ) = 54	( $\beta\gamma$ ) = 21	( $\gamma$ ) = 75
Total	(AB) = 64	(A $\beta$ ) = 30	(A) = 94	( $\alpha$ B) = 28	( $\alpha\beta$ ) = 15	( $\alpha$ ) = 43	(B) = 92	( $\beta$ ) = 45	N= 137

[41] Fill the following table:

$$(ABC) = 15; (AB\gamma) = 25; (A\beta C) = 22; (\alpha BC) = 31; (A\beta\gamma) = 8; (\alpha B\gamma) = 12;$$

$$(\alpha\beta C) = 15; (\alpha\beta\gamma) = 9$$

**Solution:-**

	A			$\alpha$			Total		
	B	$\beta$	Total	B	$\beta$	Total	B	$\beta$	Total

C	(ABC) =15	(AβC) =22	(AC) =37	(αBC) =31	(αβC) =15	(αC)= <b>46</b>	(BC) =46	(βC) =37	(C) =83
γ	(ABγ) =25	(Aβγ) =8	(Aγ)= 33	(αBγ) =12	(αβγ) =9	(αγ)= <b>21</b>	(Bγ) =37	(βγ) =17	(γ) =54
Total	(AB) =40	(Aβ) =30	(A) =70	(αB) =43	(αβ) =24	(α) =67	(B) =83	(β) =54	N =137

$$(BC) = (ABC) + (\alpha BC)$$

$$(BC) = 15 + 31 = 46$$

$$(\beta C) = (A\beta C) + (\alpha\beta C)$$

$$(\beta C) = 22 + 15 = 37$$

$$(B\gamma) = (AB\gamma) + (\alpha B\gamma)$$

$$(B\gamma) = 25 + 12 = 37$$

$$(\beta\gamma) = (A\beta\gamma) + (\alpha\beta\gamma)$$

$$(\beta\gamma) = 8 + 9 = 17$$

[42] Complete the following table:

$$(ABC) = 25; (AB\gamma) = 15; (A\beta C) = 18; (\alpha BC) = 23; (A\beta\gamma) = 13; (\alpha B\gamma) = 28;$$

$$(\alpha\beta C) = 21; (\alpha\beta\gamma) = 30$$

**Solution:-**

	A			α			Total		
	B	β	Total	B	β	Total	B	β	Total
C	(ABC) =25	(AβC) =18	(AC) =43	(αBC) =23	(αβC) =21	(αC)= 44	(BC) =48	(βC) =39	(C) =87

$\gamma$	$(AB\gamma)$ = 15	$(A\beta\gamma)$ = 13	$(A\gamma)$ = 28	$(\alpha B\gamma)$ = 28	$(\alpha\beta\gamma)$ = 30	$(\alpha\gamma)$ = 58	$(B\gamma)$ = 43	$(\beta\gamma)$ = 43	$(\gamma)$ = 86
Total	$(AB)$ = 40	$(A\beta)$ = 31	$(A)$ = 71	$(\alpha B)$ = 51	$(\alpha\beta)$ = 51	$(\alpha)$ = 102	$(B)$ = 91	$(\beta)$ = 82	$N$ = 173

$$(BC) = (ABC) + (\alpha BC)$$

$$(BC) = 25 + 23 = 48$$

$$(\beta C) = (A\beta C) + (\alpha\beta C)$$

$$(\beta C) = 18 + 21 = 39$$

$$(B\gamma) = (AB\gamma) + (\alpha B\gamma)$$

$$(B\gamma) = 15 + 28 = 43$$

$$(\beta\gamma) = (A\beta\gamma) + (\alpha\beta\gamma)$$

$$(\beta\gamma) = 13 + 30 = 43$$

[43] Fill the following table:

$$(ABC) = 50; (AB\gamma) = 72; (A\beta C) = 33; (A\beta\gamma) = 21; (\alpha BC) = 22; (\alpha B\gamma) = 30;$$

$$(\alpha\beta C) = 10; (\alpha\beta\gamma) = 16$$

	A			$\alpha$			Total		
	B	$\beta$	Total	B	$\beta$	Total	B	$\beta$	Total
C	$(ABC)$ = 50	$(A\beta C)$ = 33	$(AC)$ = 83	$(\alpha BC)$ = 22	$(\alpha\beta C)$ = 10	$(\alpha C)$ = 32	$(BC)$ = 72	$(\beta C)$ = 43	$(C)$ = 115
$\gamma$	$(AB\gamma)$ = 72	$(A\beta\gamma)$ = 21	$(A\gamma)$ = 93	$(\alpha B\gamma)$ = 30	$(\alpha\beta\gamma)$ = 16	$(\alpha\gamma)$ = 46	$(B\gamma)$ = 102	$(\beta\gamma)$ = 37	$(\gamma)$ = 139

Total	(AB) = 102	(Aβ) = 54	(A) = 176	(αB) = 52	(αβ) = 26	(α) = 78	(B) = 174	(β) = 80	N = 254
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$$(BC) = (ABC) + (\alpha BC)$$

$$(BC) = 50 + 22 = 72$$

$$(\beta C) = (A\beta C) + (\alpha\beta C)$$

$$(\beta C) = 33 + 10 = 43$$

$$(B\gamma) = (AB\gamma) + (\alpha B\gamma)$$

$$(B\gamma) = 72 + 30 = 102$$

$$(\beta\gamma) = (A\beta\gamma) + (\alpha\beta\gamma)$$

$$(\beta\gamma) = 21 + 16 = 37$$

[44] Fill the following table:

$$(ABC) = 32; (AB) = 125; (AC) = 72; (BC) = 60; (A) = 224; (B) = 301;$$

$$(C) = 150; N = 800$$

	A			α			Total		
	B	β	Total	B	β	Total	B	β	Total
C	(ABC) = 32	(AβC) = 40	(AC) = 72	(αBC) = 28	(αβC) = 50	(αC) = 78	(BC) = 60	(βC) = 90	(C) = 150
γ	(ABγ) = 93	(Aβγ) = 59	(Aγ) = 152	(αBγ) = 148	(αβγ) = 350	(αγ) = 498	(Bγ) = 241	(βγ) = 409	(γ) = 650

Total	(AB) = 125	(Aβ) = 99	(A) = 224	(αB) = 176	(αβ) = 400	(α) = 576	(B) = 301	(β) = 499	N = 800
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[45] Information is given below:

A is the player plays cricket

B is the player plays hockey

C is the player plays football

Player plays cricket = (A) = 105;

Player plays hockey = (B) = 126;

Player plays football = (C) = 75;

Play both cricket and hockey = (AB) = 24;

Play both cricket and football = (AC) = 30;

Play hockey and football = (BC) = 21;

Play all three games = (ABC) = 9;

Total no. of players = N = 240

(i) Find the players playing only two games

(ii) Find the players playing at least two games

(iii) Find the players playing only one game

**Solution:-**

	A			α			Total		
	B	β	Total	B	β	Total	B	β	Total
C	(ABC) = 9	(AβC) = 21	(AC) = 30	(αBC) = 12	(αβC) = 33	(αC) = 45	(BC) = 21	(βC) = 54	(C) = 75

$\gamma$	$(A\beta\gamma)$ = 15	$(A\beta\gamma)$ = 60	$(A\gamma)$ = 75	$(\alpha B\gamma)$ = 90	$(\alpha\beta\gamma)$ = 0	$(\alpha\gamma)$ = 90	$(B\gamma)$ = 105	$(\beta\gamma)$ = 60	$(\gamma)$ = 165
Total	$(AB)$ = 24	$(A\beta)$ = 81	$(A)$ = 105	$(\alpha B)$ = 102	$(\alpha\beta)$ = 33	$(\alpha)$ = 135	$(B)$ = 126	$(\beta)$ = 114	$N$ = 240

(i) Find the players playing only two games

$$= (\alpha BC) + (A\beta C) + (AB\gamma)$$

$$= 12 + 21 + 15 = 48$$

(ii) Find the players playing at least two games

$$= (\alpha BC) + (A\beta C) + (AB\gamma) + (ABC)$$

$$= 12 + 21 + 15 + 9 = 57$$

(iii) Find the players playing only one game

$$(A\beta\gamma) + (\alpha B\gamma) + (\alpha\beta C)$$

$$60 + 90 + 33 = 183$$

[47] Information is given below

A is the student failed in Marathi

B is the student failed in English

C is the student failed in Hindi

Student failed in Marathi  $= (A) = 220$ ;

Student failed in English  $= (B) = 300$ ;

Student failed in Hindi  $= (C) = 150$ ;

No. of students failed in both Marathi and English  $= (AB) = 125$ ;

No. of students failed in both Marathi and Hindi  $= (AC) = 70$ ;

No. of students failed in both English and Hindi  $= (BC) = 60$ ;

No. of students failed in all three subjects  $= (ABC) = 30$ ;

Total no. of students  $= N = 800$

(i) Find the number of students who failed in two languages

- (ii) Find the number of students who failed in only one language  
 (iii) Find the number of students who failed in at least one language

**Solution:-**

	A			$\alpha$			Total		
	B	$\beta$	Total	B	$\beta$	Total	B	$\beta$	Total
C	(ABC) =30	(A $\beta$ C) = 40	(AC) = 70	( $\alpha$ BC) = 30	( $\alpha\beta$ C) = 50	( $\alpha$ C)= 80	(BC) = 60	( $\beta$ C) =90	(C) = 150
$\gamma$	(A $B\gamma$ ) = 95	(A $\beta\gamma$ ) =55	(A $\gamma$ )= 150	( $\alpha$ B $\gamma$ ) = 145	( $\alpha\beta\gamma$ ) = 355	( $\alpha\gamma$ )= 500	(B $\gamma$ ) = 240	( $\beta\gamma$ ) = 410	( $\gamma$ ) = 650
Total	(AB) = 125	(A $\beta$ ) = 95	(A) =220	( $\alpha$ B) = 175	( $\alpha\beta$ ) = 405	( $\alpha$ ) =580	(B) = 300	( $\beta$ ) =500	N = 800

- (i) Find the number of students who failed in two languages

$$= (\alpha BC) + (A\beta C) + (AB\gamma)$$

$$= 30 + 40 + 95 = 165$$

- (ii) Find the number of students who failed in only one language

$$(A\beta\gamma) + (\alpha B\gamma) + (\alpha\beta C)$$

$$55 + 145 + 50 = 250$$

- (iii) Find the number of students who failed in at least one language

$$N - (\alpha\beta\gamma) =$$

$$800 - 355 = 445$$

[47] Information is given below:

Number of women interviewed is 170.

Number of women coming from Pune is 95.

Number of women coming from Outskirts is 75.

Among the married women coming from Pune, 25 were experienced and 10 were inexperienced.



Among the married women coming from outskirts, 8 were experienced and 60 were inexperienced.

5 women were unmarried, inexperienced staying in outskirts.

120 women were inexperienced.

Hence, find number of women from Pune who were unmarried and experienced.

**Solution:-** A be the women coming from Pune;

B be the women is married;

C be the women is experienced.

Number of experienced married women coming from Pune =  $(ABC) = 25$ ;

Number of inexperienced married women coming from Pune =  $(AB\gamma) = 10$ ;

Number of experienced married women coming from outskirts =  $(\alpha BC) = 8$ ;

Number of inexperienced married women coming from outskirts =  $(\alpha B\gamma) = 60$ ;

Number of inexperienced women =  $(\gamma) = 120$ ;

Number of inexperienced unmarried women coming from outskirts =  $(\alpha\beta\gamma) = 5$ ;

Number of women coming from Pune is  $95 = (A) = 95$ ;

Number of women coming from Outskirts is  $75 = (\alpha) = 75$ ;

$N = 170$

**Solution:-**

	A			$\alpha$			Total		
	B	$\beta$	Total	B	$\beta$	Total	B	$\beta$	Total

C	(AB) = 25	(AβC) = 15	(AC) = 40	(αBC) = 8	(αβC) = 2	(αC)= 10	(BC) = 33	(βC) = 17	(C) = 50
γ	(ABγ) = 10	(Aβγ) =45	(Aγ)= 55	(αBγ) = 60	(αβγ)= 5	(αγ)= 65	(Bγ) = 70	(βγ) =50	(γ)= 120
Total	(AB) = 35	(Aβ) = 60	(A) = 95	(αB)= 68	(αβ)= 7	(α)=75	(B)= 103	(β)= 67	N= 170

[48] Information is given below:

$(ABC) = 50$ ;  $(AB\gamma) = 75$ ;  $(A\beta C) = 40$ ;  $(\alpha BC) = 30$ ;  $(A\beta\gamma) = 20$ ;  $(\alpha B\gamma) = 30$ ;

$(\alpha\beta C) = 15$ ;  $(\alpha\beta\gamma) = 10$ . Find (B), (AC) and  $(\alpha\beta)$

$$(AB) = (ABC) + (AB\gamma)$$

$$(AB) = 50 + 75 = 125$$

$$(\alpha B) = (\alpha BC) + (\alpha B\gamma)$$

$$(\alpha B) = 30 + 30 = 60$$

$$(B) = (AB) + (\alpha B)$$

$$(B) = 125 + 60 = 185$$

$$(AC) = (ABC) + (A\beta C)$$

$$(AC) = 50 + 40 = 90$$

$$(\alpha\beta) = (\alpha\beta C) + (\alpha\beta\gamma)$$

$$(\alpha\beta) = 15 + 10 = 25$$

[49] Information is given below:

$(ABC) = 50$ ;  $(AB\gamma) = 72$ ;  $(A\beta C) = 33$ ;  $(\alpha BC) = 22$ ;  $(A\beta\gamma) = 21$ ;  $(\alpha B\gamma) = 30$ ;

$(\alpha\beta C) = 10$ ;  $(\alpha\beta\gamma) = 16$ . Find (A), (AB), (AC), (BC) and  $(A\beta)$

**Solution:-**

$$(AB) = (ABC) + (AB\gamma)$$

$$(AB) = 50 + 72 = 122$$

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$$(A\beta) = (A\beta C) + (A\beta\gamma)$$

$$(A\beta) = 33 + 21 = 54$$

$$(AC) = (ABC) + (A\beta C)$$

$$(AC) = 50 + 33 = 83$$

$$(BC) = (ABC) + (\alpha BC)$$

$$(BC) = 50 + 22 = 72$$

$$(A) = (AB) + (A\beta)$$

$$(A) = 122 + 54 = 176$$

[50] Information is given below:

$$N = 100; (A) = 65; (B) = 55; (C) = 45; (AB) = 25; (AC) = 20; (BC) = 10;$$

$$(ABC) = 8. \text{ Find } (AB\gamma), (A\gamma), (\alpha B) \text{ and } (\beta C)$$

$$(AB) = (ABC) + (AB\gamma)$$

$$25 = 8 + (AB\gamma)$$

$$(AB\gamma) = 17$$

$$(A) = (A\gamma) + (AC)$$

$$65 = (A\gamma) + 20$$

$$(A\gamma) = 45$$

$$(B) = (AB) + (\alpha B)$$

$$55 = 25 + (\alpha B)$$

$$(\alpha B) = 30$$

$$(C) = (BC) + (\beta C)$$

$$45 = 10 + (\beta C)$$

$$(\beta C) = 35$$

### Miscellaneous examples:

[1] In a biological experiment seventy fruits on a tree were examined. The

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following data was submitted by the observer:

Seedless fruits 40; Seedless and round 20; Seedless and oblong 103, Round 2.

Is the data consistent?

**Solution:** Let A denote seedless and B denote roundness. Then  $\alpha$  and  $\beta$  will have the opposite meanings. The data can be written as,

$$(AB) = 20; (A\beta) = 10; (A) = 40; N = 70;$$

$$(A) = (AB) + (A\beta)$$

$$\therefore 40 \neq 20 + 10 \neq 30$$

The data is inconsistent

**[A] Choose correct alternative from the following**

[1] A qualitative characteristic is called

- (a) Constant
- (b) Variable
- (c) Attribute
- (d) Association

**Answer:- (c) Attribute**

[2] If an attribute has two classes, it is called

- (a) dichotomy
- (b) trichotomy
- (c) classification
- (d) manifold classification

**Answer:- (a) dichotomy**

[3] If an attribute has more than two classes, it is called

- (a) dichotomy
- (b) trichotomy
- (c) classification
- (d) manifold classification

**Answer:- (d) manifold classification**

[4] The  $\chi^2$  test should not be used if any expected frequency is

- (a) less than 5
- (b) less than 10
- (c) equal to 5
- (d) more than 5

**Answer:- (a) less than 5**

[5] The total of all frequencies n is of order

- (a) Zero (b) One  
(c) Two (d) Three

**Answer:- (a) Zero**

[6] In case of consistent data, no class frequency can be

- (a) Positive (b) Negative  
(c) Both (a) and (b) (d) Neither (a) and (b)

**Answer:- (b) Negative**

[7] With two attributes A and B. the total number of ultimate frequencies is

- (a) Two (b) Four  
(c) Six (d) Nine

**Answer:- (b) Four**

[8] If  $(AB) = \frac{(A) \times (B)}{N}$  then the two attributes A and B are:

- (a) Independent (b) Dependent  
(c) Correlated (d) Quantitative

**Answer:- (a) Independent**

[9] To calculate the level of association we can calculate coefficient of association (Q), the coefficient of association always lies between

- (a) -1 and +1 (e) -1 and 0  
(c) 0 and 5 (b) 0 and 1

**Answer:- (a) -1 and +1**

[10] If the class frequency  $(AB) = 0$ , the value of Q is equal to

- (a) -1 (b) +1  
(c) 0 (d)  $\infty$

**Answer:- (a) -1**

[11] If the class frequency  $(\alpha B) = 0$ , the value of Q will be.

- (a) -1 (b) +1  
(c) 0 (d)  $\infty$

**Answer:- (b) +1**

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[12] If for two attributes the class frequency  $(AB) = (\alpha B) = 0$ , the value of the coefficient of association is

- (a) -1 (b) +1  
(c) 0 (d)  $\infty$

**Answer:- (c) 0**

[13] If for two attributes the class frequencies  $(AB)(\alpha\beta) = (A\beta)(\alpha B)$  then Q is equal to

- (a) -1 (b) +1  
(c) 0 (d)  $\infty$

**Answer:- (c) 0**

[14] If two attributes A and B are independent then the coefficient of association is

- (a) -1 (b) +1  
(c) 0 (d)  $\infty$

**Answer:- (c) 0**

[15] If for two attributes A and B the class frequencies  $(AB)(\alpha\beta) = (A\beta)(\alpha B)$  then A and B are called

- (a) Independent (b) Positively associated  
(c) Negatively associated (d) All of the above

**Answer:- (a) Independent**

[16] If for two attributes A and B the class frequencies  $(AB)(\alpha\beta) > (A\beta)(\alpha B)$  then A and B are called

- (a) Independent (b) Positively associated  
(c) Negatively associated (d) All of the above

**Answer:- (b) Positively associated**

[17] If for two attributes A and B the class frequencies  $(AB)(\alpha\beta) < (A\beta)(\alpha B)$  then A and B are called.

- (a) Independent (b) Positively associated

(c) Negatively associated

(d) All of the above

**Answer:- (c) Negatively associated**

[18] If  $(AB) < \frac{(A) \times (B)}{N}$  the association between two attributes A and B is

(a) Negative

(b) Positive

(c) Zero

(d) Symmetrical

**Answer:- (a) Negative**

[19] If  $(AB) > \frac{(A) \times (B)}{N}$  the association between two attributes A and B is

(a) Negative

(b) Positive

(c) Zero

(d) Symmetrical

**Answer:- (b) Positive**

[20] If two attributes A and B have perfect positive association then the value of coefficient of association is equal to

(a) -1

(b) +1

(c) 0

(d)  $\infty$

**Answer:- (b) +1**

[21] A characteristic which varies in quality from one individual to another is called

(a) Variable

(b) Constant

(c) Attribute

(d) Statistic

**Answer:- (c) Attribute**

[22] The eyes colour of 50 women is:

(a) Variable

(b) Constant

(c) Attribute

(d) Discrete

**Answer:- (c) Attribute**

[23] The frequency of class can always be expressed as a sum of frequency:

(a) Lower order classes

(b) Higher order classes

(c) Zero order classes

(d) None of the above

**Answer:- (b) Higher order classes**

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## EXERCISE:

### THEORY QUESTIONS

- [1] Define attribute and give illustrations.
- [2] Distinguish between a variable and an attribute.
- [3] Explain the following terms:
  - (i) dichotomy, (ii) positive attribute, (iii) negative attribute, (iv) order of a class- frequency, (v) ultimate class-frequency, (vi) independence of two attributes, (vii) positive association between two attributes, (viii) dissociation between two attributes.
- [4] Distinguish between association and correlation.
- [5] What do you understand by consistency of data.
- [6] Write a short note on association of attribute.

### NUMERICAL EXAMPLES:

- [1] From the given data find out the missing frequencies  $(AB) = 100$ ,  $(A) = 300$ ,  $(N) = 1000$ ,  $(B) = 600$
- [2] From the following ultimate class frequencies find positive and negative classes and the total number of observations from a nine square table.  
 $(AB) = 100$ ,  $(\alpha B) = 80$ ,  $(A\beta) = 40$
- [3] In a group of 200 persons, 142 read Marathi newspaper, 108 read English newspaper and 85 read both the papers. Find the number of persons reading
  - (i) only Marathi newspaper
  - (ii) newspaper in only one language
  - (iii) no newspaper
- [4] Out of 110 students interview for their liking in music 48 students liked Indian music. 73 liked Western music and 20 liked both. Find the number of students who did not like music.
- [5] A teacher examined 280 students in Economics and Auditing and found that 160 failed in Economics, 140 failed in Auditing and 80 failed in both the subjects, is there any association between failure in Economics and Auditing ?



[6] In a co-education Institute, out of 200 students 150 were boys. They took an examination and it was found that 120 passed, 10 girls had failed, is there any association between sex and success in the examination.

[7] From the following data, find the association between darkness of eye-colour in father and son;

Father with dark eyes and sons with dark eyes-140

Father with dark eyes and sons with not dark eyes-160

Father with not dark eyes and sons with dark eyes-180

Father with not dark eyes and sons with not dark eyes-120

What would have been the frequency of father with dark eyes and sons with dark eyes for the same total number had there been complete independence?

[8] Investigate the association between eye colour of wives from the data given below.

Husbands with light eyes and wives with light eyes=309

Husbands with light eyes and wives with non- light eyes =214

Husbands with non-light eyes &wives with light eyes=132

Husbands with non-light eyes &wives with non-light eyes=119

[9] In an assorted study to find whether tall husband tend to marry tall wives, the following information about the wives of 1250 tall and 1250 short statured husband was published find the coefficient of association between the stature of the wives and husband.

	Tall husbands	short husbands
Tall wives	60%	10%
Short wives	10%	50%

[10] Do you find any association between the temperaments of brother and sisters from the following data :

Good natured brother and good natured sisters	1230
Good natured brother and sullen sisters	850
Sullen brother and good sisters	530
Sullen brother and sullen sisters	980

[11] Among 252 candidates, 140 were boys, 72 candidates were successful ,among them 40 were boys . Obtain coefficient of association and comment on result

[12] In an institute out of 200 students 150 were boys. In an examination 120 boys and 40 girls passed. Find Yule’s coefficient of association between sex and success and comment on the result:

[13] Out of 200 students that appeared for M.B. A. examination, 80 were married. Among 60 students who failed, 24 were married. Find the coefficient of association between marriage and failure in the examination. Comment on your result:

[14] 400 students, 160 were married . Among 120 students who failed, 48 were married. Find coefficient of association between attributes marriage and failure.

[15] Calculate Yule’s coefficient of association between the weight of the children and their respective economic condition.

		Poor children	Rich children
Below	normal	75	23
weight			
Above	normal	5	42
weight			

[16] Calculate Yule's coefficient of association between the inoculation and not attacked by hepatitis

	Not attacked	Attacked
Inoculated	431	5
Not inoculated	291	9

[17] 400 saplings were planted, 210 got flowered . Fertilizers were applied to 100 of which 85 got flowered . Discuss whether application of fertilizers is useful in flowering of plants

[18] Complete the table:

$$(ABC) = 15; (AB\gamma) = 25; (A\beta C) = 22; (\alpha BC) = 31; (A\beta\gamma) = 8; (\alpha B\gamma) = 12;$$

$$(\alpha\beta C) = 15; (\alpha\beta\gamma) = 9$$

[19] Complete the table:

$$(ABC) = 25; (AB\gamma) = 15; (A\beta C) = 18; (\alpha BC) = 23; (A\beta\gamma) = 13; (\alpha B\gamma) = 28;$$

$$(\alpha\beta C) = 21; (\alpha\beta\gamma) = 30$$

[20] Complete the table:

$$(ABC) = 50; (AB\gamma) = 72; (A\beta C) = 33; (A\beta\gamma) = 21; (\alpha BC) = 22; (\alpha B\gamma) = 30;$$

$$(\alpha\beta C) = 10; (\alpha\beta\gamma) = 16$$

[21] Information is given below

A is the student failed in Marathi

B is the student failed in English

C is the student failed in Hindi

$$\text{Student failed in Marathi} = (A) = 220;$$

$$\text{Student failed in English} = (B) = 300;$$

$$\text{Student failed in Hindi} = (C) = 150;$$

$$\text{No. of students failed in both Marathi and English} = (AB) = 125;$$

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No. of students failed in both Marathi and Hindi =  $(AC) = 70$ ;

No. of students failed in both English and Hindi =  $(BC) = 60$ ;

No. of students failed in all three subjects =  $(ABC) = 30$ ;

Total no. of students =  $N = 800$

(i) Find the number of students who failed in two languages

(ii) Find the number of students who failed in only one language

(iii) Find the number of students who failed in at least one language

[22] Information is given below:

Number of women interviewed is 170.

Number of women coming from Pune is 95.

Number of women coming from Outskirts is 75.

Among the married women coming from Pune, 25 were experienced and 10 were inexperienced. Among the married women coming from outskirts, 8 were experienced and 60 were inexperienced. 5 women were unmarried, inexperienced staying in outskirts. 120 women were inexperienced.

Hence, find number of women from Pune who were unmarried and experienced.

[23] Information is given below:

$(ABC) = 50$ ;  $(A\beta\gamma) = 72$ ;  $(A\beta C) = 33$ ;  $(\alpha BC) = 22$ ;  $(A\beta\gamma) = 21$ ;  $(\alpha B\gamma) = 30$ ;

$(\alpha\beta C) = 10$ ;  $(\alpha\beta\gamma) = 16$ . Find  $(A)$ ,  $(AB)$ ,  $(AC)$ ,  $(BC)$ .

[24] As from the following data find out whether attributes are independent associated or disassociated

i)  $(AB)$ , ii)  $(AB)$ , iii)  $(\alpha B)$  and iv)  $(\alpha B)$ ;  $N = 100$ ,  $(A) = 40$ ,  $(B) = 80$ ,  $(AB) = 30$