PAPER-II

CHAPTER-1

Poisson Distribution:

Poisson distribution was discovered by the French mathematician and physicist Denis Poisson (1781-1840)who published it 1837 [Poisson distribution is a limiting case of the binomial distribution under the following condition]

(i) n be the number of trials is indefinitely large i.e. $n \rightarrow \infty$

(ii) P the constant probability of each trials is indefinitely small

(iii) $np = \lambda$ is finite

$$\therefore p = \frac{\lambda}{n} \text{ and } q = 1 - \frac{\lambda}{n}$$

Where, λ is positive real number.

Uses of Poisson distribution (Real life situation)

[i] Every discrete distribution where the sample size is highly large and probability of the random variable is considerable small.

[ii] Number of deaths from a discrete (not in the from of an epidemic) such as heart attack or cancer or due to snake bite, COVID-19.

[iii] Number of faulty blade in a packet of 100.

[iv] Number of suicides reported in a particular city.

[v] Number of air accident in some unit of time.

[vi] Number of printing mistake at each page of the book.

[vii] Number of telephone call connection to number in a telephone exchange.

[viii] No. of transgender birth occurrence in a family of Maharashtra.

[ix] No. of Divyangajan birth occurrence in a family of Maharashtra.

[x] Six sixes in over of a specific one day.

Definition:-

A random variable X is said to follow a Poisson distribution if it assumed only non-negative values and as probability mass function is given by-

$$P(x,\lambda) = P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}; & x=0,1,2......;\lambda > 0 \\ 0 & =\text{otherwise}; \end{cases}$$

X is Poisson variate/ distributed with parameter λ and it is denoted by $X \sim P(\lambda)$

Remark:-

$[1]\ \infty! = 1$ $\qquad [2]\ \dfrac{\infty}{\infty} = \infty$ $\qquad [3]\ \lambda^{\infty} = \infty$

$$[4]\ e^{\lambda} = \left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + ... + \frac{\lambda^{\infty}}{\infty!}\right] = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + ... + \frac{\lambda^{\infty}}{\infty!}$$

$$[5]\ \sum_{x=0}^{\infty} P(X=x) = e^{-\lambda}\sum_{x=0}^{\infty}\frac{\lambda^x}{x!} = e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + ... + \frac{\lambda^{\infty}}{\infty!}\right] = e^{-\lambda} \times e^{\lambda} = 1$$

$[6]\ \left(1-\dfrac{\lambda}{n}\right)^{n} = e^{-\lambda}$ $\qquad [7]\ \left(1-\dfrac{\lambda}{n}\right)^{x} = 1$

$$[8]\ \text{Log } (1-a) = -\left(a + \frac{a^2}{2} + \frac{a^3}{3} + \frac{a^4}{4} + ...\right) \qquad ;\text{if } |a| < 1$$

Reduction of binomial to Poisson distribution

Proof: Let $b(x;n,p)$ be the binomial distribution

$$b(x;n,p) = {}^{n}c_{x}\ p^{x}\ q^{n-x}$$

$$b(x;n,p) = {}^{n}c_{x}\ p^{x}\ (1-p)^{n-x}$$

$$b(x;n,p) = \frac{n!}{x!(n-x)!}\left(\frac{\lambda}{n}\right)^{x}\left(1-\frac{\lambda}{n}\right)^{n-x} \qquad \left(\because p = \frac{\lambda}{n}\right)$$

$$b(x;n,p)=\frac{\lambda^x}{x!}\ \frac{n!}{(n-x)!n^x}\left(1-\frac{\lambda}{n}\right)^n.\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$b(x;n,p)=\frac{\lambda^x}{x!}\ \frac{n(n-1)(n-2).....(n-x+1)(n-x)!}{n^x\ (n-x)!}\ \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x}$$

$$b(x;n,p)=\frac{\lambda^x}{x!}\ e^{-\lambda}\ \frac{n(n-1)\ (n-2).......(n-x+1)}{n^x} \qquad \left(\because \left(1-\frac{\lambda}{n}\right)^n=e^{-\lambda}\ \&\ \left(1-\frac{\lambda}{n}\right)^x=1\right)$$

$$b(x;n,p)=\frac{\lambda^x}{x!}e^{-\lambda}\ \frac{n}{n}\left(\frac{n-1}{n}\right)\ \left(\frac{n-2}{n}\right).......\left(\frac{n-x+1}{n}\right)$$

$$b(x;n,p)=\frac{e^{-\lambda}\lambda^x}{x!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right).......\left(1-\frac{x-1}{n}\right)$$

Taking limit

$$b(x;n,p)=\frac{e^{-\lambda}\lambda^x}{x!}\lim_{n\to\infty}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right).......\left(1-\frac{x-1}{n}\right)$$

But,

$$=\lim_{n\to\infty}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\left(1-\frac{3}{n}\right).......\left(1-\frac{x-1}{n}\right)=1$$

$$=\frac{\lambda^x}{x!}\times 1\times\frac{e^{-\lambda}}{1}$$

$$=\frac{e^{-\lambda}\lambda^x}{x!}$$

$$\therefore \lim b(x,n,p)=\frac{e^{-\lambda}\lambda^x}{x!}$$

Mean and Variance of Poisson distribution

$$\text{Mean} = E(X) = \sum x\, p(x)$$

$$\text{Mean} = E(X) = \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$\text{Mean} = E(X) = e^{-\lambda} \sum_{x=0}^{\infty} \frac{x\lambda^{x}}{x!}$$

$$\text{Mean} = E(X) = e^{-\lambda}\left[\frac{0}{0!} + \frac{\lambda}{1!} + \frac{2\lambda^{2}}{2!} + \frac{3.\lambda^{3}}{3!} + ... + \frac{\infty.\lambda^{\infty}}{\infty!}\right]$$

$$\text{Mean} = E(X) = e^{-\lambda}\left[\frac{0}{0!} + \frac{\lambda}{1!} + \frac{2\lambda^{2}}{2!} + \frac{3.\lambda^{3}}{3!} + ... + \infty\right] \qquad \left(\because \frac{\infty.\lambda^{\infty}}{\infty!} = \infty\right)$$

$$\text{Mean} = E(X) = e^{-\lambda}\lambda\left[1 + \lambda + \frac{3.\lambda^{2}}{3\times 2!} + ...\right]$$

$$\text{Mean} = E(X) = e^{-\lambda}\lambda\left[1 + \lambda + \frac{3.\lambda^{2}}{3\times 2!} + ...\right]$$

$$\text{Mean} = E(X) = e^{-\lambda}\lambda\, e^{\lambda} \qquad \left(\because e^{\lambda} = 1 + \lambda + \frac{\lambda^{2}}{2!} + \frac{\lambda^{3}}{3!} + ... + \frac{\lambda^{\infty}}{\infty!}\right)$$

$$\text{Mean} = E(X) = \lambda.e^{-\lambda+\lambda}$$

$$\text{Mean} = E(X) = \lambda.e^{0}$$

$$\text{Mean} = E(X) = \lambda\times 1$$

$$\text{Mean} = E(X) = \lambda$$

Mean of Poisson distribution is λ

Variance = $\mu_2 = E(x^2) - [E(x)]^2 .....(i)$

Now,

$$E(x^2) = \sum x^2 P(x)$$

$$E(x^2) = \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$E(X^2)=\sum_{x=0}^{\infty}[x^2+x-x]\frac{e^{-\lambda}\lambda^x}{x!}$$

$$E(X^2)=\sum_{x=0}^{\infty}[x(x-1)+x]\frac{\lambda^x}{x!}\left[e^{-\lambda}\right]$$

$$E(X^2)=\left[e^{-\lambda}\right]\sum_{x=0}^{\infty}x(x-1)\frac{\lambda^x}{x!}+\sum_{x=0}^{\infty}x\frac{e^{-\lambda}\lambda^x}{x!}$$

$$E(X^2)=\left[e^{-\lambda}\right]\left[0(-1)\frac{\lambda^0}{0!}+1\times\frac{0\times\lambda^1}{1!}+2\times1\frac{\lambda^2}{2!}+3\times2\frac{\lambda^3}{3!}+\frac{4\times3}{4!}-\lambda^4\right]+\lambda$$

$$E(X^2)=\left[e^{-\lambda}\right]\left[0+0+\lambda^2+\frac{3\times2}{3\times2\times1}\lambda^3+\frac{4\times3}{4\times3\times2\times1}\lambda^4+......\right]+\lambda$$

$$E(X^2)=\left[e^{-\lambda}\right]\left[\lambda^2+\lambda^3+\frac{\lambda^4}{2\times1}+.........\right]+\lambda$$

$$E(X^2)=\left[e^{-\lambda}\lambda^2\right]\left[1+\lambda+\frac{\lambda^2}{2!}+.....\right]+\lambda$$

$$E(X^2)=e^{-\lambda}\lambda^2e^{\lambda}+\lambda$$

$$E(X^2)=\lambda^2+\lambda$$

Put this value in equation (i), we get

$$\text{variance}=\mu_2=\lambda^2+\lambda-\lambda^2$$

$$\text{variance}=\mu_2=\lambda$$

Mean and variance of Poisson distribution is equal

Moment Generating Function of the Poisson Distribution:

$$M_X(t)=\sum e^{tx}P(x)$$

$$M_X(t)=\sum_{x=0}^{\infty}\frac{e^{tx}e^{-\lambda}\lambda^x}{x!}$$

$$M_X(t)=\sum_{x=0}^{\infty}\frac{e^{-\lambda}(\lambda e^t)^x}{x!}$$

$$M_X(t)=e^{-\lambda}\left(\frac{(\lambda e^t)^0}{0!}+\frac{(\lambda e^t)^1}{1!}+\frac{(\lambda e^t)^2}{2!}+........\right)$$

$$M_X(t)=e^{-\lambda}\left(1+\frac{(\lambda e^t)}{1!}+\frac{(\lambda e^t)}{2!}+........\right)$$

$$M_X(t)=e^{\lambda(e^t-1)}$$

$$M_X(t)=e^{-\lambda}e^{\lambda e^t} \qquad \left(\because e^{\lambda e^t}=1+\frac{\lambda e^t}{1!}+\frac{(\lambda e^t)^2}{2!}+....\right)$$

$$M_X(t)=e^{\lambda(e^t-1)}$$

Additive or reproductive property:

Statement:

Sum of two independent Poisson variates X_1 and X_2 is also Poisson variate with parameter $\lambda_1+\lambda_2$.

Proof: X_1 and X_2 are two independent Poisson variate with parameters $\lambda_1 \& \lambda_2$

$$M_{X_1}(t) = e^{\lambda_1(e^t-1)} \quad \& \quad M_{X_2}(t)=e^{\lambda_2(e^t-1)}$$

$$M_{X_1+X_2}(t) = M_{X_1}(t).M_{X_2}(t) \qquad (\because X_1 \text{ and } X_2 \text{ are independent})$$

$$M_{X_1+X_2}(t) = e^{\lambda_1(e^t-1)}.e^{\lambda_2(e^t-1)}$$

$$M_{X_1+X_2}(t) = e^{(\lambda_1+\lambda_2)(e^t-1)}$$

$$x_1+x_2 \sim P(\lambda_1+\lambda_2)$$

Cumulant generating function (C.G.F.) of the Poisson distribution:

If $X \sim P(\lambda)$ then the m. g. f. is

$$M_X(t) = e^{\lambda(e^t-1)}$$

Hence, the cumulant generating function (c.g.f.) will be,

$$K_X(t) = \log_e M_X(t)$$

$$K_X(t) = \log_e\left[e^{\lambda(e^t-1)}\right]$$

$$K_X(t) = \lambda\left(e^{\lambda}-1\right)$$

$$K_X(t) = \lambda\left[\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\ldots+\frac{t^r}{r!}+\ldots\right)-1\right]$$

$$K_X(t) = \lambda\left[t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\ldots+\frac{t^r}{r!}+\ldots\right]$$

Now,

$$K_r = r^{th}\text{cumulant} = \text{coefficient of } \frac{t^r}{r!} = \lambda \qquad \forall r$$

i.e. $K_1=K_2=K_3=K_4=\cdots=\lambda$

Thus, we have proved all cumulant of Poisson distribution are equal and the common value is parameter λ. This is regarded as characteristic property of Poisson distribution. It means a Poisson distribution satisfies this property and conversely any discrete probability distribution satisfying this property must be Poisson distribution.

Central moments using first four cumulant are obtained as follows:

$$\mu_1=K_1=\lambda$$

$$\mu_2=K_2=\lambda$$

$$\mu_3=K_3=\lambda$$

$$\mu_4=K_4+3K_2^2=\lambda+3\lambda^2$$

Note: Obtaining central moments of Poisson r. v. using cumulant is the easiest method.

Coefficients of skewness and kurtosis:

Since, $\mu_3 = \lambda > 0$, Poisson distribution is positively skewed.

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\lambda^2+\lambda}{\lambda^2} = 3+\frac{1}{\lambda} > 3$$

$$\gamma_2 = \beta_2 - 3 = \frac{1}{\lambda} > 0$$

Hence, Poisson distribution is leptokurtic. Further as $\lambda \to \infty$, $\beta_1, \gamma_1, \gamma_2 \to 0$ and $\beta_2 \to 3$.

Thus, for large λ, the distribution becomes symmetric and mesokurtic.

- **Mode of the Poisson distribution**

Let

$P[X = x]$ is the maximum

$$P\left[X = (x-1)\right] \le P(X = x) \ge P\left[X = (x+1)\right]$$

If x is maximum, $x-1$ & $x+1$ is minimum

$$\frac{e^{-\lambda}\lambda^{(x-1)}}{(x-1)!} \le \frac{e^{-\lambda}\lambda^{x}}{x!} \ge \frac{e^{-\lambda}\lambda^{(x+1)}}{(x+1)!}$$

Multiplying by $\frac{x!}{e^{-\lambda}\lambda^{x}}$

$$\frac{e^{-\lambda}\lambda^{(x-1)}}{(x-1)!} \times \frac{x!}{e^{-\lambda}\lambda^{x}} \le \frac{e^{-\lambda}\lambda^{x}}{x!} \times \frac{x!}{e^{-\lambda}\lambda^{x}} \ge \frac{e^{-\lambda}\lambda^{(x+1)}}{(x+1)!} \times \frac{x!}{e^{-\lambda}\lambda^{x}}$$

$$\frac{x}{\lambda} \le 1 \ge \frac{\lambda}{x+1}$$

Where,

$$\frac{x}{\lambda} \leq 1$$

$$x \leq \lambda$$

Again

$$1 \geq \frac{\lambda}{x+1}$$

$$(x+1) \geq \lambda$$

$$\lambda \leq (x+1)$$

$$\lambda - 1 \leq x$$

$$\therefore \lambda - 1 \leq x \leq \lambda$$

Case I: When λ is an integer, the distribution is bimodal and we have

$$\therefore \quad P(X = \lambda - 1) = P(X = \lambda)$$

Since, x is always integral in Poisson distribution. Which shows that distribution is bi-modal.

Case-(II): When λ is not an integer, we have the value of x as the greatest integer within λ

Probability distribution of sum of two Geometric variables:

Statement: If X_1 and X_2 are two independent and identically distributed (i.i.d.) random variables having geometric distribution with parameter P then $x_1 + x_2$ does not follow geometric distribution. In other words, additive property does not hold good for geometric distribution.

Proof: As X_1 and X_2 are i.i.d. with geometric distribution with parameter P.

$$P(X_1 = r) = pq^r, \quad r = 0,1,2,\ldots\ldots; 0 < p < 1$$

And

$$P(X_2 = r) = pq^r, \quad r = 0,1,2,\ldots\ldots; 0 < p < 1$$

Suppose, $Z = X_1 + X_2 = n$

Then p.m.f. of Z is given by

$$\begin{aligned}
P(Z = n) &= P(X_1 + X_2 = n) \\
P(Z = n) &= \sum_{r=0}^{n} P(X_1 = r, X_2 = n-r) \\
P(Z = n) &= \sum_{r=0}^{n} P(X_1 = r)P(X_2 = n-r) \quad (\because X_1 \text{ and } X_2 \text{ are two independent}) \\
P(Z = n) &= \sum_{r=0}^{n} pq^r pq^{n-r} \\
P(Z = n) &= \sum_{r=0}^{n} p^2 q^n \quad (\because p^2 q^n \text{ is constant}) \\
P(Z = n) &= (n+1)p^2 q^n
\end{aligned}$$

Which is not p.m.f. of geometric distribution

[1] If X is Poisson variate such that P(X=2)=9 P(X=4)+ 90P(X=6) Find

(i) λ (ii) mean of X

Solution:

If X is a Poisson variable with parameter λ then

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}\quad ; X=0,1,2,\ldots\ldots\ldots\ldots\ldots\lambda>0$$

But, P(X=2) = 9 P(X=4)+ 90P(X=6)

$$\frac{e^{-\lambda}\lambda^{2}}{2!}=9e^{-\lambda}\left[\frac{\lambda^{4}}{4\times3\times2}+10\frac{\lambda^{6}}{6\times5\times4\times3\times2\times1}\right]$$

$$\frac{\lambda^{2}}{2}=\frac{9}{4\times3\times2}\times\lambda^{2}\left[\lambda^{2}+\frac{10\lambda^{4}}{6\times5}\right]$$

$$1=\frac{3}{4}\left(\lambda^{2}+\frac{\lambda^{4}}{3}\right)$$

$$1=\frac{3}{4}\lambda^{2}+\frac{\lambda^{4}}{4}$$

$$1 = \frac{3\lambda^2}{4} + \frac{\lambda^4}{4}$$

$$4 = 3\lambda^2 + \lambda^4$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

$$\lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0$$

$$\lambda^2(\lambda^2+4) - 1(\lambda^2+4) = 0$$

$$(\lambda^2-1)(\lambda^2+4) = 0$$

$$\Rightarrow \lambda = 1, -1, -4$$

$$\Rightarrow \lambda = 1$$

(ii) Mean of X = $\lambda = 1$

[2] If X and Y are independent Poisson variate such that P(X=1)=P(X=2) and P(Y=2)=P(Y=3). Find the variance of X -2Y

Solution: Let X and Y are Poisson distribution with parameters λ_1 and λ_2

$$\therefore X \sim P(\lambda_1)$$

$$\therefore Y \sim P(\lambda_2)$$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,3........,\lambda_1 > 0$$

$$P(Y = y) = \frac{e^{-\lambda_2}\lambda^y}{y!}, \quad y = 0,1,2,3........,\lambda_2 > 0$$

But given, P(X=1) = P(X=2)

$$\frac{e^{-\lambda_1}\lambda_1^1}{1!} = \frac{e^{-\lambda_1}\lambda_1^{\ 2}}{2!}$$

$$\lambda_1 = \frac{\lambda_1 \times \lambda_1}{2}$$

$$2 = \lambda_1$$

$$\lambda_1 = 2$$

And

$$P(Y=2) = P(Y=3)$$

$$\frac{e^{-\lambda}\lambda_2^2}{2!} = \frac{e^{-\lambda_2}\lambda_2^3}{3!}$$

$$\frac{\lambda_2^{\ 2}}{2\times1} = \frac{\lambda_2^{\ 3}}{3\times2\times1}$$

$$1 = \frac{\lambda^2}{3}$$

$$\therefore \lambda_2 = 3$$

$$\text{Var}\left(x\text{-}2y\right) = 1^2\text{Var}\left(x\right) + 2^2\text{Var}\left(y\right) \text{-}2\ \text{Cov}\left(x,y\right)$$

X & Y are independent $\therefore$ $\text{Cov}\left(x,y\right) = 0$

$= \text{Var}\left(x\right) + 4\text{Var}\left(y\right)$

$= 2 + 4\times3$ ($\because$ mean & variance are equal)

$= 14$

[3] If X & Y are two independent r. v. Such that P(X=2) = P(X=3) & P(Y=3) = P(Y=4). Find the variance of (2X – 3Y).

Solution: $\lambda_1 = 3$ and $\lambda_2 = 4$, var $\left(2x\ \text{-}3y\right)$ = 48

[4] If for Poisson variate, $E(X^2)=20$ find the E(X) & Var(X) .

Solution:-For Poisson variate with a parameter $\lambda > 0$

$$\text{Var}\left(X\right) = E\left(x^2\right) - \left[E\left(X\right)\right]^2 = E\left(x^2\right) \text{-}\lambda^2$$

Now,

$E\left(x^2\right) = \lambda^2+\lambda$

$20 = \lambda^2+\lambda$

$\lambda^2+\lambda\text{-}20 = 0$

$\lambda^2+5\lambda\text{-}4\lambda\text{-}20 = 0$

$\lambda\left(\lambda+5\right)\text{-}4\left(\lambda+5\right)= 0$

$\left(\lambda+5\right)\left(\lambda\text{-}4\right)= 0$

$\left(\lambda+5\right)=0 \quad \text{or} \quad \left(\lambda\text{-}4\right)=0$

$\lambda=-5 \quad \text{or} \quad \lambda=4$

$\therefore \lambda=4, \quad \lambda>0$

$E\left(x\right) = \text{mean} = \lambda = 4$

$E\left(x\right)= 4$

$Var\left(x\right) = 4$ ($\because$ mean & variance are same)

[5] If X and Y are independent Poisson variate such that P(X=1) = P(X=2) and P(Y=2) = P(Y=3). Find the variance of X -2Y

Solution: Let X and Y are distributed poissonaly with parameter λ_1 and λ_2

$\therefore X \sim P(\lambda_1)$

$\therefore Y \sim P(\lambda_2)$

$$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,3........,\lambda_1>0$$

$$P(Y=y)=\frac{e^{-\lambda_2}\lambda^y}{y!}, \quad y = 0,1,2,3........,\lambda_2>0$$

But given, P(X=1) = P(X=2)

$$\frac{e^{-\lambda_1}\lambda_1^1}{1!}=\frac{e^{-\lambda_1}{\lambda_1}^2}{2!}$$

$$\lambda_1=\frac{\lambda_1\times\lambda_1}{2}$$

$$2=\lambda_1$$

$$\lambda_1=2$$

And $P(Y=2)=P(Y=3)$

$$\frac{e^{-\lambda}\lambda_2^2}{2!}=\frac{e^{-\lambda_2}\lambda_2^3}{3!}$$

[6] For a Poisson distribution P(X=1) = 0.03 and P(X=2) = 0.2, find P(X=0) and P(X=3)

Solution: We know that,

If $X \sim P(\lambda)$

Recurrence Relation is,

$$P[X=x+1]=\frac{\lambda}{x+1}P[X=x]; \qquad x = 0,1,2,3,......$$

$$P[X=2]=\frac{\lambda}{2}P[X=1]$$

$$0.2=\frac{\lambda}{2}0.03$$

$$\frac{0.4}{0.03}=\lambda$$

$$\lambda=\frac{40}{3}$$

Now,

$$P[X=1]=\frac{\lambda}{1}P[X=0]$$

$$0.03=\frac{40}{3}P[X=0]$$

$$P\left[X=0\right] = 0.0023$$

Also,

$$P[X=3]=\frac{\lambda}{3}P[X=2]$$

$$P[X=3]=\frac{40}{3}\times\frac{1}{3}\times 0.2$$

$$P[X=3]=\frac{8}{9}=0.8889$$

[7] Let X be Poisson variate with parameter λ.If $P[X=5]=\frac{3}{10}P[X=4]$,find $P(X > 3)$

Solution: By recurrence relation between probabilities.

$$P[X=x+1]=\frac{\lambda}{x+1}P[X=x]$$

$$P[X=4+1]=\frac{\lambda}{4+1}P[X=4]$$

$$P[X=5]=\frac{\lambda}{5}P[X=4] \qquad \cdots(1)$$

But given,

$$P[X=5]=\frac{3}{10}P[X=4]$$

$$\frac{\lambda}{5}P[X=4]=\frac{3}{10}P[X=4]$$

$$\frac{\lambda}{5}=\frac{3}{10}$$

$$\lambda=\frac{3}{2}$$

$$\lambda=1.5$$

Hence, p.m.f. is given by.

$$P[X=x]=\frac{e^{-1.5}(1.5)^x}{x!}, \qquad x=0,1,2,3,.......$$

[8] A personal officer knows that about 20% of the applicants for a certain position are suitable for the job. What is the probability that the 5th person interviewed will be the first one who is suitable?

Solution: Let

X: Number of candidates interviewed for selecting the first suitable candidate.

p = probability that the candidate will be selected = 0.2

Here X has geometric distribution with parameter p = 0.2

$\therefore$ The p.m.f. is given by,

$$P(X=x)= pq^x \qquad x = 0,1,2,3\ldots; \ q=1-p$$

$$P(X=x)= pq^{x-1} \qquad x = 1,2,3\ldots; \ q=1-p$$

$$P(X=5)= pq^{5-1}$$
$$P(X=5)= pq^4$$
$$P(X=5)=(0.2)(1-0.2)^4$$
$$P(X=5)= 0.08192$$

[9] If the probability that a certain test yields a positive reaction is equal to 0.4. What is probability that less than 5 negative reaction occur before the first positive one?

Solution: Let,

X: Number of negative reaction before first positive one.

P = probability that a certain test yields positive reaction = 0.4

$\therefore$ X has geometric distribution with parameter P = 0.4

In this case p. m. f. is given by,

$$P(X=x) = pq^x \qquad x = 0,1,2,3\ldots; \ q=1-p$$

$$P(X<5) = P(X \le 4)$$

$$P(X<5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$P(X<5) = p + pq + pq^2 + pq^3 + p\,q^4$$

$$P(X<5) = p(1 + q + q^2 + q^3 + q^4)$$

$$P(X<5) = p\frac{(1-q^5)}{(1-q)}$$

$$P(X<5) = p\frac{(1-q^5)}{p}$$

$$P(X<5) = (1-q^5)$$

$$P(X<5) = (1-(0.6)^5)$$

$$P(X<5) = 1 - 0.07776$$

$$P(X<5) = 0.92224$$

[10] Number of road accidents on a high way during a month follows a Poisson distribution with mean 5. Find the probability that a certain month number of accidents on the highway will be

i) Less than 3

ii) Between 3 and 5

iii) More than 3

Solution:

Let x denote number of road accidents on a highway during a months and

λ=mean=5

$\therefore x \sim P(\lambda)$

$\therefore$ the p.m.f. is

$$P[X=x]=\frac{e^{-5}5^{x}}{x!}; \quad x=0,1,2,......$$

(i) $P[X<3]=P[X \le 2]$

$$P[X<3]=P[X \le 2]=P[X=2]+P[X=1]+P[X=0]$$

$$P[X<3]=P[X \le 2]=\frac{e^{-5}5^{2}}{2!}+\frac{e^{-5}5^{1}}{1!}+\frac{e^{-5}5^{0}}{0!}$$

$P[X<3]=P[X \le 2]$=from statistical tables

$$P[X<3]=P[X \le 2]=0.006738+0.033690+0.089224$$

$$P[X<3]=P[X \le 2]=0.124652$$

(ii) $P[3 \le X \le 5]=P[x=3]+P[x=4]+P[x=5]$

$$P[3 \le X \le 5]=\frac{e^{-5}\lambda^{3}}{3!}+\frac{e^{-5}\lambda^{4}}{4!}+\frac{e^{-5}\lambda^{5}}{5!}$$

$P[3 \le X \le 5]$=from statistical tables

$$P[3 \le X \le 5]=0.140374+0.175467+0.175467$$

$$P[3 \le X \le 5]=0.491308$$

(iii) $P[X>3]=1-P[X \le 3]$

$P[X>3]=1-\left(P[X=0]+P[X=1]+P[X=2]+P[X=3]\right)$

$$P[X>3]=1-\left\{\frac{e^{-5}\lambda^0}{0!}+\frac{e^{-5}\lambda^1}{1!}+\frac{e^{-5}\lambda^2}{2!}+\frac{e^{-5}\lambda^3}{3!}\right\}$$

P[X>3]=from statistical tables

$P[X>3]=1-\{0.006738+0.033690+0.084224+0.14037\}$

$P[X>3]=1-0.2650$

$P[X>3]=0.7350$

Que. 1 Fill in the blanks and complete the following statements:

[15] $X \to$ B (n,p) tends to poisson (m) distribution if ,

a. $n \to \infty, p \to 1/2$ b. $n \to 0, p \to \infty$

c. $n \to 100, p \to 0$ d. $n \to \infty, p \to 0, np = m < \infty$

[16] The second central moment of poisson distribution with mean m is……

a. m b. 3m c. m^2 d. m^3

[17] Let X_1 and X_2 be independent poisson variates : $Z = X_1 + X_2$ as ……distribution .

a. binomial b. poisson c. geometric d. negative binomial

[18] $X \to$ poisson (m) then ,

a. mean = variance b. mean > variance

c. mean < variance d. mean = standard deviation

[19] If X ~> poisson (m) then its MGF is

a. $e^{m(t-1)}$ b. $e^{m(e^t-1)}$

c. $e^{m(e^t+1)}$ d. $e^{(me^t-1)}$

[20] If $X \rightarrow$ poisson (m) with variance 3,then the third cumulant is

a. $\sqrt{3}$ b. 3^2 c. 3 d. 1/3

(C) . Answer in brief:

[21] If $M_x(t) = e^{2.4(e^t-1)}$ is MGF of r.v. x, then identify the probability distribution of x

[22] comment on mode of poisson distribution when its mean is integer

[23] If X ~> poisson (m) such that p(X=z) =3/4 p(X=1), find the mode of

[24] If X ~> poisson (m) such that $\beta_1 = 0.5$ then find p(X<1)

[25] If all the cumulants of a r. v. x are equal to k,find μ_4

[26] If X ~> poisson (m), such that p(X=5)=3/10 p(X=4), find p(X=0)

[27] If X ~> poisson (m), such that p(X=1)=2p(X=2), find the mean of X

[28] If X ~> poisson (m) with modes at X =4 and X=5 , then find the p(X=0)

(D). state true or false

[29] If X and Y are independent poisson r.v.s then X –Y is also a poisson random variable.

[30] poisson distribution is always unimodal

[31] poisson distribution is symmetric.

[32] poisson distribution satisfies additive property.