INTEREST AND ANNUITY

Introduction:-

Borrower: The person who takes the loan is called as borrower.

Lender: The person or any organization who gives the loan is called as lender.

Principal: The total amount of money borrowed is called the principal.

Rate of interest: The rate of interest is the amount charged for the use of the principal for a given length of time, usually on yearly or per annum basis. It is denoted by r.

Time: The period of time for which the money is borrowed or invested. It is denoted by n.

Interest: Additional money paid by the borrowed to the lender for using the money is called interest.

Simple Interest: If the interest is calculated uniformly on the original principal throughout the lone period, it is called simple interest. It is denoted by S. I.

Amount: The amount is the sum of the principal and interest. It is denoted by A.

The following points should be taken care of while solving problems on simple interest:

[1] Unless, stated otherwise, (i) interest implies simple interest, (ii) rate of interest

[2] When the particular month is not mentioned, then one has to consider one month to be consisting of 30 days and 12 months comprise one year.

[3] 1 year consists of 365 days. In case, the year is a leap year then the month of 1. February consists of 29 days and the year consists of 366 days. [4] In order to find the interest from one date to another, only one terminal date is to be included and the other is to be excluded. For example, the number of days from 9 June to 30 June = 21

[5] Rate of interest is r% per annum then $i = \frac{r}{100}$ = rate of interest on a rupee for a year.

[6] The symbol @ implies 'at the rate of and p.a. stands for 'per annum'.

General Formula to calculate Simple Interest

If P denotes the principal (Rs.), r denotes the rate (percentage p.a.) and n denotes time (years), then interest is

[i] $I = \frac{P \times n \times r}{100}$

If the denotes the amount, then A = P + I

[ii] $A = P + \frac{P \times n \times r}{100} = P\left(1 + \frac{n \times r}{100}\right)$

Illustrative Examples

[1] Find the simple interest and amount on Rs.900 for 3 years 4 months at 5% per annum.

Solution:

P = Rs.900; r = 5% p.a.

n = 3years and 4 months = $\left(\frac{40}{12}\right)$ years $= \left(\frac{10}{3}\right)$ years

we know,

$$I = \frac{P \times n \times r}{100}$$

$$I = \frac{900 \times \frac{10}{3} \times 5}{100}$$

$$I = 3 \times 10 \times 5$$

$$I = \text{Rs.}150$$

Amount = P + I = Rs.900 + Rs.150 = Rs.1050

[2] Find the simple interest and amount on Rs.1000 for 6 months at 4% per annum.

Solution:

P = Rs.1000; r = 4% p.a. ; n = 6 months = $\left(\frac{6}{12}\right)$ years = $\left(\frac{1}{2}\right)$ years

we know,

$$\text{S. I.} = \frac{P\times n\times r}{100}$$

$$\text{S. I.} = \frac{1000\times\frac{1}{2}\times 4}{100}$$

$$\text{S. I.} = 10\times 2$$

S. I. = Rs.20

Therefore, A = P + S.I. = (1000 + 20) = Rs.1020

[3] Find the simple interest on Rs.5000 for 146 days at $\left(15\frac{1}{2}\right)$% per annum.

Solution:

P = Rs.5000; r = $15\frac{1}{2}$% p.a. = $\frac{31}{2}$% p.a ; n = 146 days = $\left(\frac{146}{365}\right)$ years

We know,

$$\text{S. I.} = \frac{P\times n\times r}{100}$$

$$\text{S. I.} = \frac{5000\times\frac{146}{365}\times\frac{31}{2}}{100}$$

$$\text{S. I.} = \frac{5000\times 146\times 31}{100\times 365\times 2}$$

$$\text{S. I.} = \frac{226300}{730}$$

S. I. = Rs.310

[4] Find the simple interest on Rs.1200 from 9th April to 21st June at 10% per annum.

Solution:

P = Rs.1200; r = 10% p.a;

n = 9th April to 21st June = 73 days n = $n=\left(\frac{73}{365}\right)$ years

$$S.\ I. = \frac{P\times n\times r}{100}$$

$$S.\ I. = \frac{1200\times\frac{73}{365}\times 10}{100}$$

$$S.\ I. = \frac{12\times 73\times 10}{365}$$

$$S.\ I. = \frac{8760}{365}$$

S. I. = Rs.24

[5] In how much time dose Rs.500 invested at the rate of 8% p.a. simple interest amounts to Rs.580.

Solution:

Here P = Rs.500; r = 8% p.a.; A = Rs.580

Therefore, S.I = A - P = (580 - 500) = Rs.80

$$S.\ I. = \frac{P\times n\times r}{100}$$

$$80 = \frac{500\times n\times 8}{100}$$

$$\frac{1}{n} = \frac{5\times 8}{80}$$

$$\frac{1}{n} = \frac{40}{80}$$

$$\frac{1}{n} = \frac{1}{2}$$

n = 2 years

[6] In how many years will a sum of Rs.400 yield an interest of Rs.132 at 11% per annum?

Solution:

P = Rs.400; r = 11% and S.I = Rs.132

$$\text{S. I.} = \frac{P\times n\times r}{100}$$

$$132 = \frac{400\times n\times 11}{100}$$

$$\frac{1}{n} = \frac{4\times 11}{132}$$

$$\frac{1}{n} = \frac{44}{132}$$

$$\frac{1}{n} = \frac{1}{3}$$

$$n = 3 \text{ years}$$

[7] In how many years will a sum double itself at 8 % per annum?

Solution:

Let Principal = P, then, Amount = 2P

So, S.I. = A - P = 2P – P = P

$$\text{S. I.} = \frac{P\times n\times r}{100}$$

$$P = \frac{P\times n\times 8}{100}$$

$$\frac{1}{n} = \frac{P\times 8}{P\times 100}$$

$$\frac{1}{n} = \frac{2}{25}$$

$$n = 12.5 \text{ years}$$

[8] In how many years will simple interest on certain sum of money at $6\frac{1}{4}\%$ Per annum be $\frac{5}{8}$ of itself?

Solution:

Let P = Rs. x, then

$$\text{S.I} = \text{Rs.}\left(\frac{5}{8}\right)x$$

$$r = 6\frac{1}{4}\% = \frac{25}{4}\%$$

$$\text{S. I.} = \frac{P\times n\times r}{100}$$

$$\left(\frac{5}{8}\right)x = \frac{x\times n\times 25}{100\times 4}$$

$$\frac{1}{n} = \frac{x\times 8}{x\times 4\times 4\times 5}$$

$$\frac{1}{n} = \frac{1}{10}$$

$$n = 10\text{ years}$$

[9] Find at what rate of interest per annum will Rs. 600 amount to Rs. 708 in 3 years.

Solution:

P= Rs.600; A = Rs.708; n = 3 years

Therefore, S.I. = Rs.708 - Rs.600 = Rs.108

$$\text{S. I.} = \frac{P\times n\times r}{100}$$

$$108 = \frac{600\times n\times r}{100}$$

$$\frac{1}{r} = \frac{6\times 3}{108}$$

$$\frac{1}{r} = \frac{1}{6}$$

$$r = 6\%\text{ p.a.}$$

[10] At what rate per cent per annum will a sum triple itself in 12 years?

Solution:

Let the sum be Rs.P, then Amount = Rs.3P

S.I. = Rs. 3P – P = Rs. 2P, n = 12 years

$$\text{S. I.} = \frac{P\times n\times r}{100}$$

$$2P = \frac{P\times 12\times r}{100}$$

$$\frac{1}{r} = \frac{P\times 12}{100\times 2P}$$

$$\frac{1}{r} = \frac{3}{50}$$

$$r = 16.67\%\ \text{p.a.}$$

Compound Interest:

When money is borrowed on simple interest then the interest is calculated uniformly on the original sum (principal) throughout the period of loan. But in everyday life, the interest changed/paid is rarely simple interest. However, in post offices, banks, insurance corporations and other companies which lend money and accept deposits, the method of calculating interest is quite different. Under this method, the borrower and the lender agree to fix a certain unit of time, say one year or a half-year or one quarter of a year (i.e., 3 months), to settle the previous account. In such cases, the interest accrued during the first unit of time is added to the original principal and the amount so obtained is taken as the principal for the second unit of time. The amount of this principal at the end of the second unit of time becomes the principal for the third unit of time and so on.

After a certain specified period, the difference between the amount and the money borrowed is called the **compound interest** (abbreviated as **CI**).

Formula for compound Interest:-

Let P is the principal and r is the rate interest per conversion period as I in decimal, n is the number of conversion period. The accrued amount after n payment periods A_n. We have the accrued amount at the end of first payment period is

$$A_1 = \text{Principal} + \text{C.I.} = P + P \times i$$
$$A_1 = P(1+i)$$

The accrued amount at the end of second payment period is

$$A_2 = A_1 + A_1 \times i = A_1(1+i)$$
$$A_2 = P(1+i)(1+i) = P(1+i)^2$$

So on for (n-1) terms

The accrued amount at the end of n payment period is

$$A_n = A_{n-1} + A_{n-1} \times i = A_{n-1}(1+i)$$
$$A_n = P(1+i)^{n-1}(1+i) = P(1+i)^n$$

$$A_n = P\left(1+\frac{r}{k \times 100}\right)^n = P(1+i)^n$$

Therefore, compound interest = **(Amount) - (Principal)**

$$\text{C. I.} = A - P = P(1+i)^n - P = P\left[(1+i)^n - 1\right] = \left\{P\left[(1+i)^n - 1\right]\right\}$$

Where,

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{k \times 100};$$

n = Total conversions = t x no. of conversions per year

k = Number of conversion periods per year

Conversion period:-

Definition:-The period at the end of which the interest is compounded is called conversion period.

When the interest is calculated and added to the principal every three months the conversion period is three months. In this case number of conversion periods per year would be four.

Total conversion periods are given below:

Conversion period	Description	Number of conversion period in a year
1 day	Compounded daily	**365**
1 month	Compounded monthly	**12**
3 months	Compounded quarterly	**4**
6 months	Compounded semi-annually	**2**
12 months	Compounded annually	**1**

Compound Interest on a given sum in the i^{th} year:

The compound interest for the n^{th} year is given by

I_n=C.I. in ‘n’ years – C.I. in (n-1) years

$$I_n = P(1+i)^n - P - \left[P(1+i)^{n-1} - P\right]$$

$$I_n = P(1+i)^n - P - P(1+i)^{n-1} + P$$

$$I_n = P(1+i)^n - P(1+i)^{n-1}$$

$$I_n = P(1+i)^{n-1}\left[(1+i) - 1\right]$$

$$i = \frac{r}{100} = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100}$$

n = Total conversions = t x no. of conversions per year

[1] Find the compound interest on Rs.5000 for 3 years at 8% per annum, compounded annually.

Solution:

Principal for the first year = Rs.5000.

Interest for the first year = Rs. S. I. = $\frac{5000 \times 8 \times 1}{100}$ = Rs.400.

Amount at the end of the first year Rs. (5000 + 400) = Rs.5400.

Principal for the second year = Rs.5400.

Interest for the second year = Rs. S. I. = $\frac{5400 \times 8 \times 1}{100}$ = Rs.432.

Amount at the end of the second year = Rs. (5400 + 432) = Rs.5832.

Principal for the third year = Rs.5832.

Interest for the third year = Rs. S. I. = $\frac{5832 \times 8 \times 1}{100}$ = Rs.466.56

Amount at the end of the third year = Rs. (5832 + 466.56) = Rs.6298.56

Therefore, compound interest = Rs. (6298.56 - 5000) = Rs.1298.56

[2] Find the compound interest on Rs.10000 for 1 year at 10% per annum, compounded half-yearly.

Solution:

Rate of interest = 10% per annum = 5% per half-year.

n = 1 year = 2 half-years

Original principal = Rs.10000.

Interest for the first half-year = Rs. S. I. = $\frac{10000 \times 5 \times 1}{100}$ = Rs.500.

Amount at the end of the first half-year = Rs.(10000 + 500) = Rs.10500.

Principal for the second half-year = Rs.10500.

Interest for the second half-year = Rs. S. I. = $\frac{10500 \times 5 \times 1}{100}$ = Rs.525.

Amount at the end of the second half-year = Rs. (10500 + 525) = Rs.11025.

Therefore, compound interest = Rs. (11025 - 10000) = Rs.1025.

[3] Dr. Manmath Lohgaonkar is invested is Rs. 5000 at the rate of 10% per annum. What will be the amount after two years if compounded

(i) Annually (ii) Semi-annually (iii) Quarterly (iv) Monthly

Solution:- We have given

P = Rs.5000,

r = 10 % per annum

n = 2 years

(i) Annually

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{10}{1 \times 100} = 0.1$$

k = Number of conversion periods per year

n = Total conversions = t x no. of conversions per year = 2 x 1=2

We have

$$A_n = P\left(1 + \frac{r}{k \times 100}\right)^n = P(1+i)^n$$

$$A_2 = 5000(1+0.1)^2$$

$$A_2 = 5000(1.1)^2$$

$$A_2 = 5000 \times 1.21$$

$$A_2 = 6050$$

(ii) Semi-annually

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{10}{2 \times 100} = 0.05$$

k = Number of conversion periods per year =2

n = Total conversions = t x no. of conversions per year = 2 x 2=4

We have

$$A_n = P\left(1+\frac{r}{k\times100}\right)^n = P(1+i)^n$$

$$A_4 = 5000(1+0.05)^4$$

$$A_4 = 5000(1.05)^4$$

$$A_4 = 5000\times1.2155$$

$$A_4 = 6077.5$$

(iii) Quarterly

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}\times100} = \frac{r}{K\times100} = \frac{10}{4\times100} = 0.025$$

k = Number of conversion periods per year =4

n = Total conversions = t x no. of conversions per year = 2 x 4=8

We have

$$A_n = P\left(1+\frac{r}{k\times100}\right)^n = P(1+i)^n$$

$$A_8 = 5000(1+0.025)^8$$

$$A_8 = 5000(1.025)^8$$

$$A_8 = 5000\times1.2184$$

$$A_8 = 6092$$

(iv) Monthly

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}\times100} = \frac{r}{K\times100} = \frac{10}{12\times100} = 0.00833$$

k = Number of conversion periods per year =12

n = Total conversions = t x no. of conversions per year = 2 x 12=24

We have

$$A_n = P\left(1+\frac{r}{k\times 100}\right)^n = P(1+i)^n$$
$$A_{24} = 5000(1+0.00833)^{24}$$
$$A_{24} = 5000(1.00833)^{24}$$
$$A_{24} = 5000\times 1.22029$$
$$A_{24} = 6101.45$$

[4] Find the amount of Rs.8000 for 3 years, compounded annually at 5% per annum. Also, find the compound interest.

Solution:

Here,

P = Rs.8000,

r = 5 % per annum

n = 3 years.

We know,

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}\times 100} = \frac{r}{K\times 100} = \frac{5}{1\times 100} = 0.05$$

k = Number of conversion periods per year =1

n = Total conversions = t x no. of conversions per year = 3 x 1=3

We have

$$A_n = P\left(1+\frac{r}{k\times 100}\right)^n = P(1+i)^n$$
$$A_3 = 8000(1+0.05)^3$$
$$A_3 = 8000(1.05)^3$$
$$A_3 = 8000\times 1.1576$$
$$A_3 = 9260.8$$

And

$$\text{C. I.} = A - P = P(1+i)^n - P = P\left[(1+i)^n - 1\right] = \left\{P\left[(1+i)^n - 1\right]\right\}$$

C. I. = 9260.8 - 8000 =1260.8

Thus, amount after 3 years = Rs.9260.8

[5] Find the amount of Rs.12000 after 2 years, compounded annually; the rate of interest being 5 % p.a. during the first year and 6 % p.a. during the second year. Also, find the compound interest.

Solution:

Here, P = Rs.12000,

r_1 = 5 % p.a. and r_2 = 6 % p.a.; n = 2 years

We know,

$$i_1 = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{5}{1 \times 100} = 0.05$$

$$i_2 = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{6}{1 \times 100} = 0.06$$

k = Number of conversion periods per year =1

n = Total conversions = t x no. of conversions per year = 2 x 1=2

We have

$$A_n = P\left(1+\frac{r_1}{k \times 100}\right)^{n_1}\left(1+\frac{r_2}{k \times 100}\right)^{n_2} = P(1+i_1)^{n_1}(1+i_2)^{n_2}$$

$$A_2 = 12000(1+0.05)(1+0.06)$$

$$A_2 = 12000(1.05)(1.06)$$

$$A_2 = 13356$$

And

$$\text{C. I.} = A - P = P(1+i)^n - P = P\left[(1+i)^n - 1\right] = \left\{P\left[(1+i)^n - 1\right]\right\}$$

C. I. = 13356 - 12000 =1356

Therefore, compound interest = Rs.1356.

[6] Find the compound interest on Rs.125000, if Anjali took loan from a bank for 12 months at 8 % per annum, compounded quarterly.

Solution:

Here,

P = Rs.125000, r = 8 % per annum

$$i = \frac{r}{k \times 100} = \frac{8}{4 \times 100} = 0.02$$

n = 12 months = 4quarters

n = Total conversions = t x no. of conversions per year = 1 x 4 =4

$$\text{C. I.} = A - P = P(1+i)^n - P = P\left[(1+i)^n - 1\right] = \left\{P\left[(1+i)^n - 1\right]\right\}$$

$$\text{C. I.} = \left\{125000\left[(1+0.02)^4 - 1\right]\right\}$$

$$\text{C. I.} = \left\{125000\left[(1.02)^4 - 1\right]\right\}$$

$$\text{C. I.} = \left\{125000[1.0824 - 1]\right\}$$

$$\text{C. I.} = \left\{125000[0.0824]\right\}$$

$$\text{C. I.} = 10300$$

Compound interest is Rs. 10300

[7] Find the compound interest on Rs.15625 for $1\frac{1}{2}$ years at 8 % per annum when compounded half-yearly.

Solution:

Here,

P = Rs.15625, r = 8 % per annum

$$i = \frac{r}{k \times 100} = \frac{8}{2 \times 100} = 0.04$$

n = Total conversions = t x no. of conversions per year = $\frac{3}{2}\text{years} \times 2 = 3$

$$\text{C. I.} = A - P = P(1+i)^n - P = P\left[(1+i)^n - 1\right] = \left\{P\left[(1+i)^n - 1\right]\right\}$$

$$\text{C. I.} = \left\{15625\left[(1+0.04)^3 - 1\right]\right\}$$

$$\text{C. I.} = \left\{15625\left[(1.04)^3 - 1\right]\right\}$$

$$\text{C. I.} = \left\{15625\left[1.1248 - 1\right]\right\}$$

$$\text{C. I.} = \left\{15625\left[0.1248\right]\right\}$$

$$\text{C. I.} = 1950$$

Compound interest is Rs. 1950

[8] Manisha deposited Rs.1000 in a bank. In return he got Rs.1331. Bank gave interest 10% per annum. How long did she keep the money in the bank?

Solution:

Let the required time be n years. Then,

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{10}{1 \times 100} = 0.10$$

k = Number of conversion periods per year =2

n = Total conversions = t x no. of conversions per year = t x 1=t

$$A_n = P\left(1 + \frac{r}{k \times 100}\right)^n = P(1+i)^n$$

$$1331 = 1000(1+0.10)^n$$

$$\frac{1331}{1000} = (1.10)^n$$

$$\frac{11 \times 11 \times 11}{10 \times 10 \times 10} = (1.10)^n$$

$$(1.10)^3 = (1.10)^n$$

By using rule of indices

$$n = 3 \quad \therefore t = 3$$

Hence, the required time is 3 years.

[9] In what time will Rs. 8000 amount to Rs. 8820 at 10% per annum interest compounded half-yearly?

Solution:

Let the required time be n years. Then,

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{10}{2 \times 100} = 0.05$$

k = Number of conversion periods per year =2

n = Total conversions = t x no. of conversions per year = t x 2 = 2t

$$A_n = P\left(1 + \frac{r}{k \times 100}\right)^n = P(1+i)^n$$

$$8820 = 8000(1+0.05)^n$$

$$\frac{8820}{8000} = (1.05)^n$$

$$1.1025 = (1.05)^n$$

$$(1.05)^2 = (1.05)^n$$

By using rule of indices

$$n = 2 \quad \therefore t = 1$$

Hence, the required time is 1 year.

[10] In what time will Rs. 16000 amount to Rs. 18522 at 10% per annum interest compounded half-yearly?

Solution:

Let the required time be n years. Then,

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{10}{2 \times 100} = 0.05$$

k = Number of conversion periods per year =2

n = Total conversions = t x no. of conversions per year = t x 2 = 2t

$$A_n = P\left(1+\frac{r}{k\times 100}\right)^n = P(1+i)^n$$

$$18522 = 16000(1+0.05)^n$$

$$\frac{18522}{16000} = (1.05)^n$$

$$1.157625 = (1.05)^n$$

$$(1.05)^3 = (1.05)^n$$

By using rule of indices

$$n = 3 \quad \therefore t = \frac{3}{2}\text{years}$$

Hence, the required time is 1.5 years.

[11] In what rate of interest investment doubles in 7 years if compounded annually? Given that $2^{\frac{1}{7}} = 1.104090$

Solution:

Let the principal is P then, $A_n = 2P$,

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}\times 100} = \frac{r}{K\times 100}$$

k = Number of conversion periods per year =1

n = Total conversions = t x no. of conversions per year = t x 2 = 2t

$$A_n = P\left(1+\frac{r}{k\times 100}\right)^n = P(1+i)^n$$

$$2P = P(1+i)^7$$

$$2 = (1+i)^7$$

$$2^{\frac{1}{7}} = (1+i)$$

$$1.104090 = (1+i)$$

$i = 1.104090 - 1$

$i = 0.104090$

$$0.104090 = \frac{r}{1 \times 100}$$

$r = 10.40\%$

Hence, the required time is 1.5 years.

[12] Find the rate percent per annum if Rs. 2,00,000 amount to Rs. 2,31,525 in $1\frac{1}{2}$ year interest being compounded half-yearly.

Solution:

We have given

P = 2,00,000 and $A_n = 2,31,525$

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100} = \frac{r}{2 \times 100}$$

k = Number of conversion periods per year =2

n = Total conversions = t x no. of conversions per year = $\frac{3}{2}$ x 2 = 3

$$A_n = P\left(1 + \frac{r}{k \times 100}\right)^n = P(1+i)^n$$

$$\frac{231525}{200000} = (1+i)^3$$

$1.157625 = (1+i)^3$

$(1.05)^3 = (1+i)^3$

$1.05 = (1+i)$

$i = 1.05 - 1$

$i = 0.05$

$$0.05 = \frac{r}{2 \times 100}$$

$r = 10\%$

Interest rate per annum is 10%

Nominal and Effective rate of Interest:-

Nominal rate of Interest:-

In transactions involving compound interest, **the stated annual rate of interest is called the nominal rate of interest.** Thus if an investment is made at 6% converted semi-annually the nominal rate of interest on this investment is 6%.

Effective rate of Interest:-

Suppose you invest Rs. 100 for a year at the rate of 6% per annum compounded semi-annually. Effective interest rate for a year will be more than 6% per annum since interest is being compounded more than once in a year.

$$\text{Effective rate } i_e = \left(1 + \frac{r}{k \times 100}\right)^K - 1 = (1 + i)^K - 1$$

$$\text{Effective rate } i_e = \left(1 + \frac{6}{2 \times 100}\right)^2 - 1$$

$$\text{Effective rate } i_e = (1 + 0.03)^2 - 1 = (1.03)^2 - 1 = (1.0609) - 1 = 0.0609$$

Thus effective rate $r_e = 100 i_e = 100 \times 0.0609 = 6.09\%$

Effective interest rate can be computed using following formula.

$$i_e = \left(1 + \frac{r}{k \times 100}\right)^K - 1 = (1 + i)^K - 1$$

Thus effective rate $r_e = 100 i_e$

Where,

[i] i_e is the effective interest rate

[ii] i is the rate of interest in decimal = $\frac{r}{k \times 100}$

[iii] k is the number of conversion periods in a year.

[Iv] r is the nominal rate.

Note:-

[i] Nominal interest rate is also defined as a stated interest rate. This interest works according to the simple interest and does not take into account the compounding periods.

[ii] Effective interest rate is the one which caters the compounding periods during a payment plan. It is used to compare the annual interest between loans with different compounding periods like week, month, year etc. In general stated or nominal interest rate is less than the effective one.

Examples:

[1] Find the effective rate equivalent to nominal rate of 6% converted quarterly.

Solution:

We have given,

r = 6% and n = 4

$$i = \frac{r}{k \times 100} = \frac{6}{4 \times 100} = 0.015$$

$$i_e = \left(1 + \frac{r}{k \times 100}\right)^K - 1 = (1 + i)^K - 1$$

$$i_e = (1 + 0.015)^4 - 1$$

$$i_e = (1.015)^4 - 1$$

$$i_e = 1.06136 - 1$$

$$i_e = 0.06136$$

$\therefore$ The effective rate percent equivalent to the nominal rate of 6% compounded quarterly = $r_e = 100 i_e = 0.0613 \times 100 = \mathbf{6.13}\%$.

[2] Find the effective rate equivalent to nominal rate of 8% converted semi-annually.

Solution:

We have given,

r = 8% and n = 2

$$i = \frac{r}{k \times 100} = \frac{8}{2 \times 100} = 0.04$$

$$i_e = \left(1 + \frac{r}{k \times 100}\right)^K - 1 = (1 + i)^K - 1$$

$$i_e = (1 + 0.04)^2 - 1$$

$$i_e = (1.04)^2 - 1$$

$$i_e = 1.0816 - 1$$

$$i_e = 0.0816$$

$\therefore$ The effective rate percent equivalent to the nominal rate of 8% compounded semiannually = $r_e = 100 i_e = 0.0816 \times 100 = 8.16\%$.

[3] Which is a better investment 4% per year compounded monthly or 4.2% per year simple interest?

Solution:

We have given,

r = 4% and n = 12

$$i = \frac{r}{k \times 100} = \frac{4}{12 \times 100} = 0.0033$$

$$i_e = \left(1 + \frac{r}{k \times 100}\right)^K - 1 = (1 + i)^K - 1$$

$$i_e = (1 + 0.0033)^{12} - 1$$

$$i_e = (1.0033)^{12} - 1$$

$$i_e = 1.0403 - 1$$

$$i_e = 0.0403$$

$\therefore$ The effective rate percent equivalent to the nominal rate of 4% compounded monthly = $r_e = 100 i_e = 0.0403 \times 100 \; = 4.03\%$.

The effective rate of interest being less than simple interest i.e. 4.03% < 4.2%,

$\therefore$ Simple interest 4.2% per year is the better investment.

[4] Dr. Manmath wants to invest Rs. 5,000 for 4 years. He may invest the amount at 10% per annum compound interest accruing at the end of each quarter of the year or he may invest it at 10.5% per annum compound interest accruing at the end of each year. Which investment will give him better return?

Solution:

We have given,

$r_1 = 10\%$ and $n = 4$

$$i = \frac{r}{k \times 100} = \frac{10}{4 \times 100} = 0.025$$

$$i_e = \left(1 + \frac{r}{k \times 100}\right)^K - 1 = (1 + i)^K - 1$$

$i_e = (1 + 0.025)^4 - 1$

$i_e = (1.025)^4 - 1$

$i_e = 1.1038 - 1$

$i_e = 0.1038$

$\therefore$ The effective rate percent equivalent to the nominal rate of 4% compounded quarterly = $r_e = 100 i_e = 0.1038 \times 100 \; = 10.38\%$.

When interest is compounded annually, then nominal rate and effective rate are same. $\therefore$ $\mathbf{r_2 = 10.5\%}$

In the second case, therefore, the effective rate of interest p.a. is 10.5%. Since 10.5 > 10.38, the second investment will give better return.

[5] To what amount will Rs. 12000 accumulate in 12 years if invested at an effective rate of 5%.

Solution:

We have given,

r = 5% and n = 12

$$i = \frac{r}{k \times 100} = \frac{5}{1 \times 100} = 0.05$$

$$A_n = P\left(1 + \frac{r}{k \times 100}\right)^n = P(1+i)^n$$

$$A_n = 12000(1+0.05)^{12}$$

$$A_n = 12000(1.05)^{12}$$

$$A_n = 12000(1.7958)$$

$$A_n = 21549.6$$

EXERCISE:

[1] Find the Principal when…..

(a) S.I. = Rs. 192 Rate = 6% per annum Time = 4 years

(b) S.I = Rs.20 Rate = 2% per annum Time = 20 month

Answer: (a) Rs.800 (b) Rs.600

[2] Find the Rate when…..

(a) Principal = Rs. 350 Time = $2^1/_2$ years S.I. = Rs. 140

(b) Principal = Rs. 9600 Time = 3 months S.I = Rs. 72

Answer: (a) 16% (b) 3%

[3] Find the Time when…..

(a) Principal = Rs. 500 Rate = 7.5% p.a S.I. = Rs. 150

(b) Principal = Rs. 700 Rate = 18% p.a. S.I. = Rs. 78

Answer: (a) 4 years (b) $^6/_7$ years

[4] Find the simple Interest and Amount

(a) Principal = Rs. 640 Rate = $12^1/_2$% Time = 6 months

(b) Principal = Rs.10000 Rate = 18% Time = 7 years

Answer: (a) Rs.40, Rs.680 (b) Rs.12600, Rs.22600

[5] Find the sum of money that amounts to Rs. 992 in 4 years at per annum.

Answer: Rs.800

[6] What sum of money will earn an interest of Rs.162 in 3 years at the rate of 12% per annum.

Answer: Rs. 450

[7] At what rate per cent annum will a sum of money double itself in 6 year?

Answer: $16\frac{2}{3}$ %

[8] In what time will a sum of money double itself at 5 % per annum?

Answer: $6\frac{2}{3}$ years

[9] Rs. 4000 were lent each to Ron and Rob at 15% per annum for $3\frac{1}{2}$ years and 5 years respectively. Find the difference in the interest paid by them.

Answer: Rs.900

[10] Rocky lends Rs.3000 to Ken at 10% per annum and the Ken lends the same sum to Mike at 12% per annum. Find Ken's gain over a period of 3 years.

Answer: Rs.180

[11] Divide Rs.1750 into two part so that simple interest on the first when deposited for 2 years at 15% per annum and that on the second when deposited for 3 years at 16% per annum in a bank add to give the total interest of Rs.624.

Answer: Rs.1200, Rs.550

[12] Find the simple interest on Rs.6000 from 16 May 2010 to 9 October 2010 at 10% per annum.

Answer: Rs.240

[13] What sum lent out at $6^1/_4$% per annum produces the same simple interest in 2 years as Rs.2100 lent out at 5% per annum produces in 16 months?

Answer: Rs.1120

[14] In how much time will S.I. on certain sum of money at $12\frac{1}{2}$% per annum be $\frac{7}{4}$ of itself?

Answer: 14 years

[15] A certain sum amounts to Rs.2200 in 2 years to Rs.2800 in 4 years at simple interest. Find the sum and the rate per cent per annum.

Answer: $18\frac{3}{4}$%

[16] A and B borrowed Rs.3000 and Rs.3500 respectively at the same rate of simple interest for 3 years. If B paid Rs.150 more interest than A, find the rate of interest per annum.

Answer: 10%

[17] Simple interest on a certain sum is $^{4}/_{9}$ of the sum. Find the rate per cent and time if both are numerically equal.

Answer: Time = $6\frac{2}{3}$ years, R = $6\frac{2}{3}$%

[18] Find the simple interest and amount in each of the following:

(a) P = Rs.1800 R = 5% T = 1 year

(b) P = Rs.2600 R = 12% T = 3 years

(c) P = Rs.3125 R = 15% T = 73 days

(d) P = Rs.5660 R = 11% T = 9 months

(e) P = Rs.180 R = 3% T = $1^{1}/_{4}$ year

Problems on simple interest:

Answer: . (a) Rs.90, Rs.1890 (b) Rs.936, Rs.3536 (c) Rs.93.75, Rs.3218.75 (d) Rs.466.95, Rs.6126.95 (e) Rs.6.75, Rs.186.75

[19] What sum would yield an interest of Rs.36 in 3 years at 3% p.a.?

Answer: Rs.400

[20] At what rate per cent per annum will Rs.250 amount to Rs.330 in 4 years?

Answer: 8%

[21] At what rate per cent per annum will Rs.400 yield an interest of Rs.78 in $1\frac{1}{2}$ years ?

Answer: 13%

[22] In what time will Rs.400 amount to Rs.512 if the simple interest is the calculated at 14% p.a.?

Answer: 2 years

[23] A sum amount to Rs.2400 at 15% simple interest per annum after 4 years. Find the sum.

Answer: Rs.1500

[24] Dr. Manmath borrowed Rs.2000 from Sam at 8% per annum. After 6 year he cleared the amount by giving Rs.2600 cash and a watch. Find the cost of the watch.

Answer: Rs.360

[25] In how many years will Rs.400 yield an interest of Rs.112 at 14% simple interest?

Answer: 2 years

[26] In how many years will Rs.12000 yield an interest of Rs.13230 at 10% simple interest?

Answer: $10\frac{1}{4}$ years

[27] In how many years will Rs.600 double itself at 10% simple interest?

Answer: $8\frac{1}{3}$ years

[28] At what rate of simple interest will Rs.5000 amount to Rs.6050 in 3 years, 4 months?

Answer: 6.3%

[29] At what rate of simple interest will the sum of money double itself in 6 years?

Answer: $16\frac{2}{3}\%$

[30] Find the simple interest at the rate of 5% p.a. for 3 years on that principal which in 4 years, 8 months at the rate of 5% p.a. gives Rs.1200 as simple interest.

Answer: Rs.771.42

[31] At what rate per cent per annum will Rs.4000 yield an interest of Rs.410 in 2 years?

Answer: $5\frac{1}{8}\%$

[32] Simple interest on a certain sum is $^{16}/_{25}$ of the sum. Find the rate per cent and time if both are numerically equal.

[Hint: (T = R), P = x, S.I. = $\frac{16}{25}x$]

Answer: 8%, 8 years

[33] Simple interest on a sum of money at the end of 5 years is $\frac{4}{5}$ of the sum itself. Find the rate per cent p.a.

Answer: 16%

[34] Find the amount and the compound interest on Rs. 2500 for 2 years at 10% per annum, compounded annually.

Answer: A = Rs.3025, C.I. = Rs.525

[35] Find the amount and the compound interest on Rs.16000 for 3 years at 5% per annum, compounded annually.

Answer: A = Rs.18522, C.I. = Rs.2522

[36] Find the difference between the simple interest and the compound interest on Rs.5000 for 2 years at 6% per annum.

Answer: Rs.18

[37] Dr. Manmath obtained a loan of Rs. 125000 from the Allahabad Bank for buying computers. The bank charges compound interest at 8% per annum, compounded annually. What amount will he have to pay after 3 years to clear the debt?

Answer: A = Rs.6615, C.I. = Rs.615

[38] Three years ago, Dr. Manmath Lohgaonkar purchased a buffalo from Kashti for Rs.11000. What payment will discharge his debt now, the rate of interest being 10% per annum, compounded annually?

Answer: A = Rs.12321, C.I. = Rs.2321

[39] Find the amount and the compound interest on Rs. 8000 for 1 year at 10% per annum, compounded half-yearly.

Answer: A = Rs.8820, C.I. = Rs.820

[40] Anjali deposited Rs.32000 in a bank, where the interest is credited quarterly. If the rate of interest be 5% per annum, what amount will she receive after 6 months?

Answer: Rs.32805

[41] Manisha took a loan of Rs.390625 from Patil Finance. If the company charges interest at 16% per annum, compounded quarterly, what amount will discharge her debt after one year?

Answer: Rs.45697

ANNUITY:

Introduction:-

In many cases you must have noted that your parents have to pay an equal amount of money regularly like every month or every year. For example payment of life insurance premium, rent of your house (if you stay in a rented house), payment of housing loan, vehicle loan etc. In all these cases they pay a constant amount of money regularly. Time period between two consecutive payments may be one month, one quarter or one year.

Sometimes some people received a fixed amount of money regularly like pension rent of house etc. In these entire cases annuity comes into the picture. When we pay (or receive) a fixed amount of money periodically over a specified time period we create an annuity.

Definition of Annuity:-

An annuity is a fixed sum paid at regular over a specified period of time.

To be called annuity if the following condition are satisfied:-

(1) Amount paid (or received) must be constant over the period of annuity

(2) Time interval between two consecutive payments (or receipts) must be the same.

Consider following tables.

Case-I		**Case-II**		**Case-III**	
Year end	Payments / Receipts in Rs.	Year end	Payments / Receipts in Rs.	Year end	Payments / Receipts in Rs.
1	4000	1	6000	1	6000
2	5000	2	6000	2	6000
3	6000	3	0	3	6000
4	5000	4	6000	4	6000
5	4000	5	6000	5	6000
6	7000	6	6000	6	6000

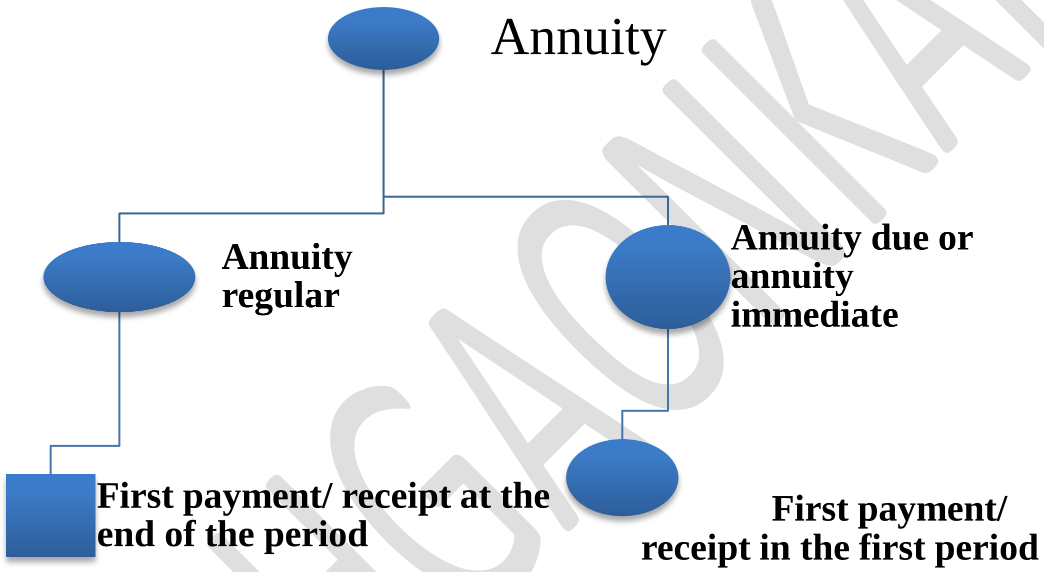
Case –I:- cannot be called annuity. Payments/Receipts though have been made at regular intervals but amount paid are not constant over the period of six years.

Case-II:- cannot be called annuity. Though amounts paid/ received are same in every year but time interval between different payments/receipts is not equal.

You may note that time interval between second and third payment/receipt is two year and time interval between other consecutive payments/receipts is only one year. You may also note that for first two year the payments/ receipts can be called annuity.

Case-II :- Can be called annuity, since, all payments/receipts over the period of six years are constant and time interval between two consecutive payments/receipts is also same i.e. one year.

Annuity regular and Annuity due / immediate:-



TYPES OF ANNUITIES:-

[i] Annuity certain:- An annuity payable for a fixed number of years is called annuity certain.

[ii] Perpetual Annuity (or Perpetuity):-An annuity which continues for ever (infinite number of years) is called perpetual annuity. Academic Prize, Nobel Prize distribution comes under perpetual annuity.

[iii] Annuity Regular or Ordinary Annuity:-. If the payments are made at the end of each period, the annuity is called an annuity regular.

The first payment is done at the end of 1^{st} year, therefore it is an annuity regular.

Year end	Payments / Receipts in Rs.
1	6000
2	6000
3	6000
4	6000
5	6000
6	6000

(iv) Annuity Due (Annuity Immediate):- If the payments are made in advance at the beginning of each period, the annuity is called annuity due or annuity immediate.

In the beginning of	Payments / Receipts in Rs.
1	6000
2	6000
3	6000
4	6000
5	6000
6	6000

FUTURE VALUE:-

Future value is the cash value of an investment at some time in the future. It is tomorrow's value of today's money compounded at the rate of interest. Suppose you invest Rs. 1,00 in a fixed deposit that pays you 7% per annum as interest. At the end of first year you will have 107. This consist of the original principal of Rs.1,000 and the interest earned of Rs.7. Rs. 107 is the future value of Rs.1,00 invested for one year at 7%. We can say that Rs.100 today is worth Rs.107 in one year's time if the interest rate is 7%.

Now suppose you invested ? 1,00 for two years. How much would you have at the end of the second year. You had Rs.1,07 at the end of the first year. If you reinvest it you end up having Rs.1,07. Rs.114.49 at the end of the second year. Thus Rs.114.49 is the future value of 1,00 invested for two years at 7%. We can compute the future value of a single cash flow by applying the formula of amount.

$$A_n = P\left(1+\frac{r}{k\times 100}\right)^n = P(1+i)^n$$

Where,

A = Accumulated amount

n = Number of conversion period

i = Rate of interest per conversion period in decimal

P = Principal

This formula can be written as by replacing A by future value F and P by single cash flow C.F.

$$F = C.F.(1+i)^n$$

Example-[1] Dr. Manmath invested Rs. 3000 at the rate of 12% per annum for two years. Calculate future value of the investment.

Solution:- We have given,

C.F. = Rs. 3000

n = 2

$$r = 12\% = \frac{12}{100} = 0.12$$

F =?

We know,

$$F = C.F.(1+i)^n$$

$$F = C.F.(1+i)^n$$

$$F = 3000(1+0.12)^2$$

$$F = 3000(1.12)^2$$

$$F = 3000(1.2544)$$

$$F = Rs.\ 3763.2$$

FUTURE VALUE OF AN ANNUITY REGULAR (AMONT OF AN ORDINARY ANNUITY):-

Future value of an annuity regular is the value, at the end of the term, of all payments. That is, it is the sum of the compound amounts of all payments.

Suppose a constant sum of Re. 1 is deposited in a savings account at the end of each year for four years at 6% interest. This implies that Re. 1 deposited at the end of the first year will grow for three years, Re. 1 at the end of second year for 2 years, Re. 1 at the end of the third year for one year and Re. 1 at the end of the fourth year will not yield any interest. Using the concept of compound interest we can compute the future value of an annuity.

The compound value of Re. 1 deposited in the first year will be.

$$A_3 = 1(1+0.06)^3 = 1(1.06)^3 = 1(1.19) = Rs.1.19$$

The compound value of Re. 1 deposited in the second year will be

$$A_2 = 1(1+0.06)^2 = 1(1.06)^2 = 1(1.12) = Rs.1.12$$

The compound value of Re. 1 deposited in the third year will be

$$A_1 = 1(1+0.06)^1 = 1(1.06) = Rs.1.06$$

The compound value of Re. 1 deposited in the fourth year will be

$$A_0 = 1(1+0.06)^0 = 1(1) = Re.1$$

The aggregate compound value of Re.1 deposited at the end of each year for four years would be:

Rs. (1.19+1.12+1.06+1.00)= Rs. 4.37

This is the compound value of an annuity of Rs. 1 for four years at 6% rate of interest. This can be written as

End of Year	Amount Deposit in Rs.	Future value at the end of fourth year Rs.
1	1	$1(1+0.06)^3 = 1.19$
2	1	$1(1+0.06)^2 = 1.12$
3	1	$1(1+0.06)^1 = 1.06$
4	1	$1(1+0.06)^0 = 1$
	Future value	4.375

The computation can be expressed as follows:

$$A(4, i) = A(1+i)^0 + A(1+i)^1 + A(1+i)^2 + A(1+i)^3$$

$$A(4, i) = A + A(1+i) + A(1+i)^2 + A(1+i)^3$$

$$A(4, i) = A\left[1 + (1+i) + A(1+i)^2 + A(1+i)^3\right]$$

Here, A is annuity and $(4, i)$ is future value at the end of year four, i is the rate of interest in decimal.

Therefore, we can extend above equation for n periods then we can write

Amount of Ist payment = $A(1+i)^{n-1}$

Amount of IInd payment = $A(1+i)^{n-2}$ and so on

Amount of (n-1)th payment = $A(1+i)^1$

Amount of (n)th payment = $A(1+i)^0 = A$

Adding these amounts in the reverse order, we get

$$A(n, i) = A(1+i)^0 + A(1+i)^1 + A(1+i)^2 + A(1+i)^3 + \ldots + A(1+i)^{n-2} + A(1+i)^{n-1}$$

$$A(n, i) = \frac{A\left[(1+i)^n - 1\right]}{(1+i) - 1}$$

$$A(n, i) = \frac{A\left[(1+i)^n - 1\right]}{i}$$

Note:- For a G.P. ,Sum of n terms $S_n = \frac{a(r^n - 1)}{r-1}$, Where, a is the first term which is A and r is common ratio which is $(1+i)$ in this G.P.

[1] Find the future value of an annuity of Rs. 500 made annually for 7 years at interest rate of 14% compounded annually. Given that $(1.14)^7 = 2.5023$.

Solution:-Here, annual payment

A = Rs. 500; n = 7; r = 14% ; $i = \frac{r}{100} = \frac{14}{100} = 0.14$

$A(7, 0.14) = ?$

We have,

$$A(n, i) = \frac{A\left[(1+i)^n - 1\right]}{i}$$

$$A(7, 0.14) = \frac{500\left[(1+0.14)^7 - 1\right]}{0.14}$$

$$A(7, 0.14) = \frac{500\left[(1.14)^7 - 1\right]}{0.14}$$

$$A(7, 0.14) = \frac{500[2.5023 - 1]}{0.14}$$

$$A(7, 0.14) = \frac{500[1.5023]}{0.14}$$

$$A(7, 0.14) = \frac{751.15}{0.14}$$

$$A(7, 0.14) = 5365.35$$

Future value of an annuity is Rs. 5365.35

[2] Find the future value of an annuity after 10 payments of Rs. 200 made annually for 10 years at interest rate of 6% compounded monthly. Given that $(1.005)^{10} = 1.0511$.

Solution:-Here, monthly payment

A = Rs. 200; n = 10; r = 6% ; $i = \frac{6}{12 \times 100} = 0.005$

$A(10, 0.005) = ?$

We have,

$$A(n, i) = \frac{A\left[(1+i)^n - 1\right]}{i}$$

$$A(10, 0.005) = \frac{200\left[(1+0.005)^{10} - 1\right]}{0.005}$$

$$A(10, 0.005) = \frac{200\left[(1.005)^{10} - 1\right]}{0.005}$$

$$A(10, 0.005) = \frac{200[1.0511 - 1]}{0.005}$$

$$A(10, 0.005) = \frac{200[0.0511]}{0.005}$$

$$A(10, 0.005) = \frac{10.22}{0.005}$$

$$A(10, 0.005) = 2044$$

Future value of an annuity is Rs. 2044

[3] Find the future value of an annuity of Rs. 600 made annually for 3 years at interest rate of 3% compounded quarterly. Given that $(1.0075)^3 = 1.0226$.

Solution:-Here, three monthly payment

A = Rs. 600 ; n = t x No. of conversion each year = 3 x 4 =12

r = 3%; $i = \frac{3}{4 \times 100} = 0.0075$

$A(12, 0.0075) = ?$

We have,

$$A(n, i) = \frac{A\left[(1+i)^n - 1\right]}{i}$$

$$A(12, 0.0075) = \frac{600\left[(1+0.0075)^{12} - 1\right]}{0.0075}$$

$$A(12, 0.0075) = \frac{600\left[(1.0075)^{12} - 1\right]}{0.0075}$$

$$A(12, 0.0075) = \frac{600[1.09380 - 1]}{0.0075}$$

$$A(12, 0.0075) = \frac{600[0.09380]}{0.0075}$$

$$A(12, 0.0075) = \frac{56.28}{0.0075}$$

$$A(12, 0.0075) = 7504$$

Future value of an annuity is Rs. 7504

FUTURE VALUE OF AN ANNUITY DUE OR ANNUITY IMMEDIATE:-

Annuity regular is the value, at the end of the term or periods of all payments. Annuity due or Annuity immediate first receipts or payments is made today. The relationship between the value of an annuity due and an ordinary annuity in case of future value is:

Future value of an Annuity due / Annuity immediate = Future value of annuity regular x$(1+i)$. i is the rate of interest in decimal

Future value of an Annuity due / Annuity immediate =

$$A(n, i) = \frac{A\left[(1+i)^n - 1\right]}{i} \times (1+i)$$

[1] Dr. Manmath Lohgaonkar invests Rs. 20000 every year starting from today for next 10 years and rate of interest 8% per annum compounded annually. Calculate future value of the annuity. Given that $(1.08)^{10} = 2.15892500$.

Solution:-Here, three monthly payment

A = Rs. 20000;   n = t x No. of conversion each year = 10 x 1 =10

r = 8%; $i = \frac{8}{1 \times 100} = 0.08$

$A(10, 0.08) = ?$

We have,

$$A(n, i) = \frac{A\left[(1+i)^n - 1\right]}{i} \times (1+i)$$

$$A(10, 0.08) = \frac{20000\left[(1+0.08)^{10} - 1\right]}{0.08} \times (1+0.08)$$

$$A(10, 0.08) = \frac{20000\left[(1.08)^{10} - 1\right]}{0.08} \times (1+0.08)$$

$$A(10, 0.08) = \frac{20000[2.15892500 - 1]}{0.08} \times (1.08)$$

$$A(10, 0.08) = \frac{20000[1.15892500]}{0.08} \times (1.08)$$

$$A(10, 0.08) = \frac{23178.5}{0.08} \times (1.08)$$

$$A(10, 0.08) = 289731.25 \times (1.08)$$

$$A(10, 0.08) = 312909.75$$

Future value of an annuity due is Rs. 312909.75

PRESENT VALUE:-

Future value is tomorrow's value of today's money compounded at the rate of interest. We can say present value is today’s value of tomorrow’s money discounted at the interest rate. Future value and present value are reciprocal of each other. Suppose you invested Rs. 1,00 in a fixed deposit that pays you 7% per annum as interest. At the end of first year you will have 107. 100 is the present value of tomorrow’s Rs. 107 at 7%.

Now suppose you invested? 100 for two years at 7% per annum we will get Rs.114.49 at the end of the second year. Thus Rs.114.49 is the future value of today’s Rs.100 invested for two years at 7%. We can compute the present value of a cash flow by applying the formula of compound interest.

The present value

$$A_n = P\left(1+\frac{r}{k\times 100}\right)^n = P(1+i)^n$$

$$P = \frac{A_n}{(1+i)^n}$$

Where,

A = Accumulated amount

n = Number of conversion period

i = Rate of interest in decimal

P = Present value

Examples:-

[1] What is the present value of Rs. 100 to be received after 2 years compounded annually at 10% interest rate?

Solution:-we have given,

$$A_n = 100;\ n = 2;\ i = \frac{10}{1\times 100} = 0.1$$

$$P = \frac{A_n}{(1+i)^n}$$

$$P = \frac{100}{(1+0.1)^2}$$

$$P = \frac{100}{(1.1)^2}$$

$$P = \frac{100}{1.21}$$

$$P = 82.64$$

Thus Rs. 82.64 shall grow to Rs. 100 after 2 years at 10% interest rate compounded annually.

[2] What is the present value of Rs. 10000 to be received after 5 years compounded annually at 12% interest rate?

Solution:-we have given,

$$A_n = 10000;\ n = 5;\ i = \frac{12}{1 \times 100} = 0.12$$

$$P = \frac{A_n}{(1+i)^n}$$

$$P = \frac{10000}{(1+0.12)^5}$$

$$P = \frac{10000}{(1.12)^5}$$

$$P = \frac{10000}{1.7623}$$

$$P = 5674.40$$

Thus Rs. 5674.40 shall grow to Rs. 10000 after 5 years at 12% interest rate compounded annually.

PRESENT VALUE OF AN ANNUITY REGULAR (ORDINARY ANNUITY):-

The present value or capital value of an annuity is the sum of the present values of payments. In other words, the present value of an annuity represents the amount of that must be invested now to purchase the payments due in the future.

The payments made at different points of time along with their present values are shown below.

End of Year	Amount Deposit in Rs.	Present value Rs. $\left[\frac{A_n}{(1+i)^n}\right]$
1	1000	$\left[\frac{1000}{(1+0.1)^1}\right]=909.09$
2	1000	$\left[\frac{1000}{(1+0.1)^2}\right]=826.44$
3	1000	$\left[\frac{1000}{(1+0.1)^3}\right]=751.31$
4	1000	$\left[\frac{1000}{(1+0.1)^4}\right]=683.01$
5	1000	$\left[\frac{1000}{(1+0.1)^5}\right]=620.92$
	Future value	3790.77

To develop a formula for the present value of an annuity regular, let us consider an annuity of five payments of Rs. 1000 each, where the interest rate per period is 10% p.a. compounded annually and the **first payment is due one period from now.**

The computation can be expressed as follows:

$$P(5,i)=\frac{A}{(1+i)^1}+\frac{A}{(1+i)^2}+\frac{A}{(1+i)^3}+\frac{A}{(1+i)^4}+\frac{A}{(1+i)^5}$$

We can extend for n periods then we get,

$$P(n,i)=\frac{A}{(1+i)^1}+\frac{A}{(1+i)^2}+\frac{A}{(1+i)^3}+\cdots+\frac{A}{(1+i)^{n-1}}+\frac{A}{(1+i)^n}$$

Here, A is annuity and (n, i) is present value at the end of year five, i is the rate of interest in decimal.

$$P(n, i) = \frac{A}{(1+i)} \frac{\left[\left(\frac{1}{(1+i)}\right)^n - 1\right]}{\left[\frac{1}{(1+i)} - 1\right]} = \frac{A}{(1+i)} \frac{\left[\frac{1}{(1+i)^n} - 1\right]}{\left[\frac{1-1+i}{(1+i)}\right]} = \frac{A}{(1+i)} \left[\frac{1-(1+i)^n}{(1+i)^n}\right]\left[\frac{(1+i)}{i}\right]$$

$$P(n, i) = \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$P(n, i) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$A = \frac{P(n, i)}{\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]}$$

Note:- For a G.P. ,Sum of n terms $S_n = \frac{a(r^n - 1)}{r-1}$, Where, a is the first term which is A and r is common ratio which is $(1+i)$ in this G.P.

PRESENT VALUE OF AN ANNUITY DUE OR ANNUITY IMMEDIATE:-

The present value or capital value of an annuity due is the sum of the present values of payments.

Formula for annuity due is = A + Present value of remaining (n-1) payments of regular annuity.

$$A + P(n, i) = A + \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

i.e.

Step 1: Compute the present value of annuity as if it were a annuity regular for one period less.

Step 2: Add initial cash payment to the step 1 value.

SINKING FUND:-

A sinking fund is a fund which is created by making equal periodical payments as to have accumulated funds to meet some financial obligation in future. For example, firm may create fund for replacing an asset at some point of time or to redeem debentures by periodically depositing a specified sum.

Therefore, if $A(n, i)$ is the targeted amount and i is the rate of interest in decimal, A be the periodic payment and n be the payment period.

Future value of an annuity regular is $A(n, i) = \dfrac{A\left[(1+i)^n - 1\right]}{i}$

Examples:

[1] Sinking fund is required to be set up for having Rs. 25,000 in 10 years. How much amount should be set aside each quarter into an account paying 6% compounded quarterly?

Solution:-

We have,

$A(40, 0.015)$ = 25,000; r = 6%; n = 4 x 10 = 40; $i = \dfrac{6}{4 \times 100} = 0.015$

Future value of an annuity regular is $A(n, i) = \dfrac{A\left[(1+i)^n - 1\right]}{i}$

We have,

$$25000 = \frac{A\left[\left(1+0.015\right)^{40}-1\right]}{0.015}$$

$$25000 = \frac{A\left[\left(1.015\right)^{40}-1\right]}{0.015}$$

$$25000 = \frac{A\left[1.8140-1\right]}{0.015}$$

$$25000 = \frac{A\left[0.8140\right]}{0.015}$$

$$25000 = 54.26$$

$$A = \frac{25000}{54.26}$$

$$A = 460.74$$

[2] Dr. Manmath Lohgaonkar plans to send his daughter for higher studies abroad after 10 years. He expects the cost of the studies to be Rs. 3000000. How much should he save at the beginning of each year to have a sum of Rs. 3000000 at the end of 10 years, if the interest rate is 8% compounded annually?

Solution:-

We have,

$A(40,\ 0.015)$ = 30, 00,000; r = 8%; n = 1 x 10 = 10;

$i = \frac{8}{1\times 100} = 0.08$ A =?

Future value of an annuity regular is

$$A(n,\ i) = \frac{A\left[\left(1+i\right)^{n}-1\right]}{i}$$

$$3000000 = \frac{A\left[\left(1+0.08\right)^{10}-1\right]}{0.08}$$

$$3000000 = \frac{A\left[\left(1.08\right)^{10}-1\right]}{0.08}$$

$$3000000 = \frac{A\left[2.1589-1\right]}{0.08}$$

$$3000000 = \frac{A\left[1.1589\right]}{0.08}$$

$$3000000 = A \times 14.48$$

$$A = \frac{3000000}{14.48}$$

$$A = 207182.32$$

Equated Monthly Installments (EMI)

Purchases on installment basis are quite common things. Now, day's costly items like home, car, refrigerator etc. are commonly purchased on installment. The repayment is generally made in monthly installments over a period of one year, ten years and twenty years etc. This monthly installment of repayment is called equated monthly installment (EMI).

There are two ways by which Banks or Housing finance companies charge interest.

[i] EMI with Reducing balance method

Dr. Manmath Lohgaonkar taken a loan of Rs. 1000000 lakhs from Central bank of India for a period of 25 years at 10 % reducing balance. In this case principal is deducted from outstanding loan amount and interest is charged on the remaining outstanding loan amount for remaining months. In details is shown in the following table.

Period	Outstanding amount	EMI =	Interest	Principal

		$X = \dfrac{P \times i}{\left[1 - \dfrac{1}{(1+i)^n}\right]}$	$I_n = P(1+i)^{n-1}(i)$	
1	1000000	9084.19	8330	754.19
2	999245.81	9084.19	8323.72	760.47
3	998485.34	9084.19	8317.38	766.81
4	997718.53	9084.19	8310.99	773.2
5	996945.33	9084.19	8304.55	779.64

$$P = \frac{X}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$X = \frac{P \times i}{\left[1 - \dfrac{1}{(1+i)^n}\right]}$$

P = Principal; r = r% p.a.;

k = Number of conversion periods per year

n = Total no. of installments = t x no. of conversions per year

X = Periodic installments

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100}$$

[i] EMI with flat rate method

In this method, the lending agency banks, housing finance companies etc. calculates the amount (principal + interest) for the given term at specified rate of interest. Then, the EMI is arrived at by dividing this amount by total number of months in the term. Thus, the method consist of

(a) Calculate amount by using formula:

$$A = P\left(1 + \frac{r\,n}{100}\right)$$

(b) $EMI = \frac{A}{k}$, Where, k = no. of months

[1] Find EMI a loan 50,000 is to be repaid in equal monthly instalments. Interest is charged at 12% p.a. on the loan outstanding at the beginning of each month and the timespan is 10 years (monthly reduction)

Solution: We have given,

P = Rs.50000;   n = 10 years;   r = 12% p.a.

k = Number of conversion periods per year =12

n = Total no. of installments = t x no. of conversions per year = 10 x 12 = 120

X = Periodic installments

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100}$$

$$i = \frac{12}{12 \times 100} = 0.01$$

We know,

$$50000 = \frac{X}{0.01}\left[1 - \frac{1}{(1+0.01)^{120}}\right]$$

$$50000 \times 0.01 = \frac{X}{0.01}\left[1 - \frac{1}{(1.01)^{120}}\right]$$

$$500 = X\left[1 - \frac{1}{3.30038}\right]$$

$$500 = X\left[1 - 0.302995\right]$$

$$500 = X\left[0.697005\right]$$

$$X = \frac{500}{0.697005}$$
$$X = 717.35$$

E.M.I. = Rs. 717.35

[2] Find EMI if a loan of 1,00,000 at the rate of 15% p.a.is to be repaid in equal monthly instalments in the span of 10 years. Interest is charged on the loan outstanding at the beginning of each years (yearly reduction)

Solution: We have given,

P = Rs.100000; n = 10 years; r = 15% p.a.

k = Number of conversion periods per year =1

n = Total no. instalments = t x no. of conversions per year = 10 x 1 = 10

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100}$$

$$i = \frac{15}{1 \times 100} = 0.15$$

We know,

$$100000 = \frac{X}{0.15}\left[1 - \frac{1}{(1+0.15)^{10}}\right]$$

$$100000 \times 0.15 = X\left[1 - \frac{1}{(1.15)^{10}}\right]$$

$$15000 = X\left[1 - \frac{1}{4.0455}\right]$$

$$15000 = X[1 - 0.247188]$$

$$15000 = X[0.752812]$$

$$X = \frac{15000}{0.752812}$$

$$X = 19925.29$$

Periodic installment is Rs. 19925.29 but we want monthly installment and therefore, Equal monthly instalments $=\frac{19925.29}{12}=1660.44$

[3] What is the EMI of loan of Rs.25,000 is repaid in 4 years. If the rate of interest is 5% p.a. on the outstanding amount at the beginning of each years.

Solution: We have given,

P = Rs.250000; n = 4 years; r = 5% p.a.

k = Number of conversion periods per year =1

n = Total no. instalments = t x no. of conversions per year = 4 x 1 = 4

$$i=\frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}\times 100}=\frac{r}{K\times 100}$$

$$i=\frac{5}{1\times 100}=0.05$$

We know,

$$25000=\frac{X}{0.05}\left[1-\frac{1}{(1+0.05)^4}\right]$$
$$25000\times 0.05=X\left[1-\frac{1}{(1.05)^4}\right]$$
$$1250=X\left[1-\frac{1}{1.215506}\right]$$
$$1250=X[1-0.822702]$$
$$1250=X[0.177298]$$
$$X=\frac{1250}{0.177298}$$
$$X=7050.27$$

Periodic installment is Rs. 7050.27 but we want monthly installment and therefore,

$$\text{Equal monthly instalments } = \frac{7050.27}{12} = 587.52$$

[4] Find EMI on a loan 100,000 is to be repaid in equal monthly installments. Interest is charged at 12% p.a. on the loan outstanding at the beginning of each month and the timespan is 2 years.

Solution: We have given,

P = Rs.100000; n = 2 years; r = 12% p.a.

k = Number of conversion periods per year =12

n = Total no. of installments = t x no. of conversions per year = 2 x 12 = 24

X = Periodic installments

$$i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year} \times 100} = \frac{r}{K \times 100}$$

$$i = \frac{12}{12 \times 100} = 0.01$$

We know,

$$100000 = \frac{X}{0.01}\left[1 - \frac{1}{(1+0.01)^{24}}\right]$$

$$100000 \times 0.01 = \frac{X}{0.01}\left[1 - \frac{1}{(1.01)^{24}}\right]$$

$$1000 = X\left[1 - \frac{1}{1.2697}\right]$$

$$1000 = X\left[1 - 0.7876\right]$$

$$1000 = X\left[0.2124\right]$$

$$X = \frac{1000}{0.2124}$$

$$X = 4708.09$$

E.M.I. = Rs. 4708.09

EXERCISE:

[1] Dr. Manmath is invested of Rs. 15000 in a Term Deposit Scheme that hatches interest 6% pa. compound quarterly. What will be the interest after one year ? What is the effective rate of interest?

Answer: Rs. 920.45 and 6.13%

[2] Find effective rate of interest corresponding to a nominal rate 3% p.a. payable (i) half yearly (ii) quarterly

Answer: (i) 3.0225% (ii) 3.033%

[3] Find EMI for a loan of 2,00.000 at the rate of 12% p.a. (reducing balance) is to be repaid in equal monthly installments in 10 years.

Answer: Rs. 2869.42

[4] Dr. Manmath borrows Rs. 8,00,000 from a finance company at 10% flat rate of interest for a period of 8 years. Find EMI.

Answer: Rs. 15000

[5] Find the difference between, EMI's by flat rate of interest and reducing balance of interest on Rs. 6,00,000 at 9% p.a. for a period of 10 years.

Answer: Rs. 1899.45

[6] Find the EMI on a loan of Rs.16,00,000 for a period of 20 years at 12% p.a. (reducing balance).

Answer: Rs. 17617.38